

A NON-TECHNICAL DESCRIPTION OF E.D.S.A.C.
HOW THE CAMBRIDGE ELECTRONIC CALCULATOR WORKS

J. Lyons & Company Ltd.
June 1949

The following pages are a copy of a booklet (20.5cm x 26cm) with the above title in the possession of Frank Land of the LEO Computers Society. This booklet is Part A of three parts, the others being Part B with more technical detail and Part C showing how a problem can be analysed for a computer. Neither of those parts is available.

Grateful thanks to Frank Land for the loan of the original booklet

CP Burton, Computer Conservation Society 15 February 2012

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PREFACE

This statement is the first part (Part A) of a description of the electronic calculating machine which has been built at Cambridge. The statement is so prepared that it can stand by itself; it gives a general idea of the way in which the machine works, and is divided into the following five sections:-

1. Name and nature of the machine.	Page 1
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3. Forms in which information appears in the machine.	12
4. Arithmetic with binary numbers.	25
5. How the machine carries out arithmetic.	41

Part B will give a more detailed description of the way in which the different organs of the machine are designed to carry out their functions.

Part C will show the arithmetical operations and representative types of clerical process that the machine can carry out; it will also show what has to be done to analyse a complete problem so that it can be given to the machine.

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A NON-TECHNICAL DESCRIPTION OF E.D.S.A.C.1. Name and Nature of the Machine.1.1. An "Electronic" Machine.

The full title of the machine to be described in these notes is the "Electronic Delay Storage Automatic Calculator". It is a machine, built in the Mathematical Laboratory at Cambridge, for making calculations at high speed.

The machine works mainly by "electronic", as opposed to electrical or mechanical means. Whereas electrical engineering is concerned normally with the flow of electricity in conductors, "electronics" employs, in addition, circuits incorporating radio valves, in which current flow is maintained by the movement in space of tiny charges of electricity, called "electrons".

1.2. The nature of information needed to define a problem given to the machine.

The information needed is of two kinds:-

- (a) the actual figures to be worked upon, which we may term the data of the problem;
- (b) the various steps or operations that must be carried out on the figures in order to produce the required answer; these are given to the machine in the form of "orders", the whole series being termed the "programme" of the problem.

Thus if the problem to be set to the machine is that of finding the average price of:-

100 lbs of Tea A at 2/- per lb

& 200 lbs of Tea B at 1/3d per lb

the data of the problem are the amounts of 100 lbs and 200 lbs at the respective prices of 2/- and 1/3d.

The programme of orders, in its simplest form, covering any problem of this type is:-

Contd....

Multiply the quantity of the first lot by its price...100 x 2/- = 200/-

" " " " " second " " " " ...200 x 1/3 = 250/-

Add these two together to obtain the total cost 450/-

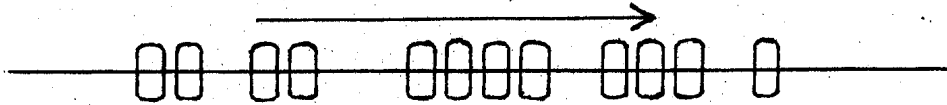
Add the two quantities together to find the total quantity 300 lbs

Divide the total cost by the total quantity to arrive at the average price 1/8d

The calculations here have been divided into a number of simple steps. The machine can solve any problem that may be similarly reduced to such a programme of orders.

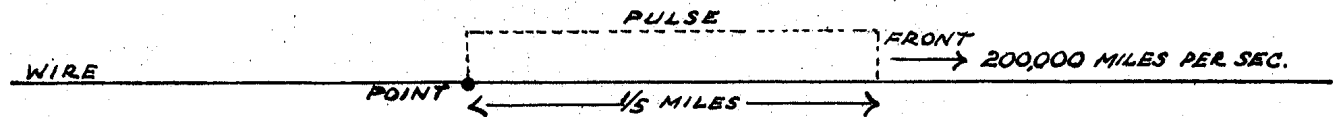
1.3. Form in which the information appears in the machine.

Both the numbers and the orders appear in the machine in the form of pulses of electricity travelling round circuits at an extremely high speed. Each different number or order is a distinctive group of pulses and gaps travelling together like a group of beads moving along a string. A single number or order might thus be represented as:-



1.4. Storage of Information.

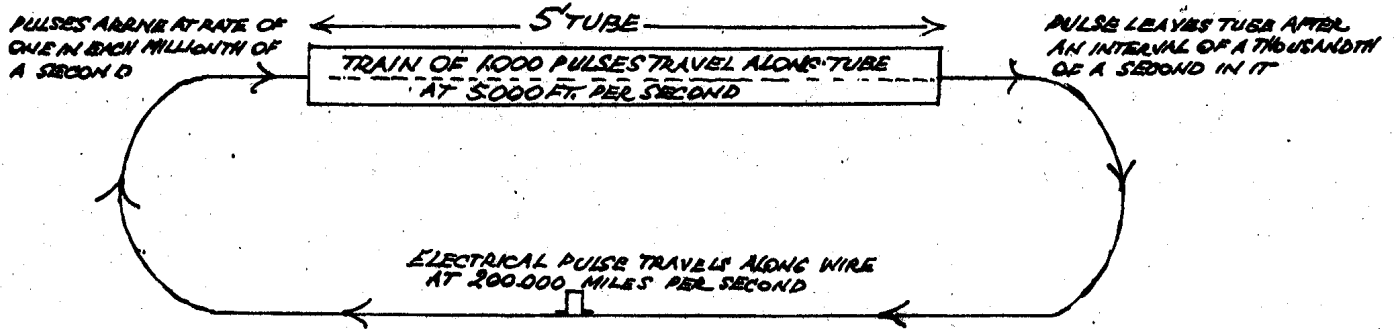
These numbers and orders can be stored by being circulated round a closed loop to be "read off" and used when required. A single pulse passing any point in the wire takes one millionth of a second to pass and it travels at a speed of nearly 200,000 miles per second. Therefore while a pulse is passing a given point - the front of it will have travelled one-fifth of a mile.



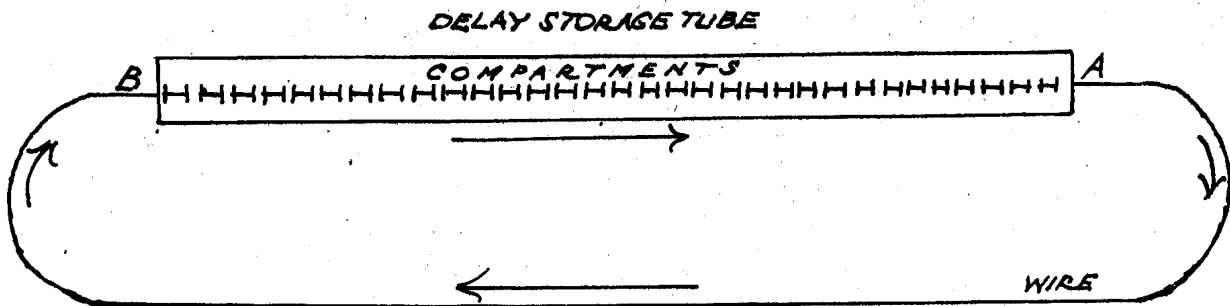
This means that if the closed loop were entirely of wire it would have to be over one-fifth of a mile long to store a single pulse, and it would be quite impracticable to store a large number of pulses in this way unless a means of reducing this speed is found. This is done in the machine by inserting in the loop a Delay circuit through which the pulses travel far more slowly.

Contd

Whereas the speed of a pulse along a wire is nearly 200,000 miles per second, the speed in the Delay circuit is less than 1 mile per second. Thus to pass along a tube about 5 feet in length (i.e. a thousandth of a mile) the pulse takes about a thousandth of a second. Now the end of the pulse arrives a millionth of a second after the front of it, so that if pulses followed one another end to end, the thousandth one would arrive before the first one left the Delay circuit.



This system therefore provides a practicable means of storing the pulses which convey the information and is called "Delay Storage", the term which is incorporated in the title of the machine. In fact each tube stores only 576 pulses because an interval of about a millionth of a second is allowed between one pulse and the next. For convenience, the train of pulses passing round the loop containing the storage tube is divided into 32 compartments, each compartment having a capacity of 18 pulses (576 in all). It must be realised that the compartments are not stationary but are moving round and round the loop about every thousandth of a second, thus:-



As the pulses included in each compartment pass along the wire A B to re-enter the Delay Storage mechanism, the information they convey may be read off for use by the machine. Also when fresh information is required to be stored, it can be put in the appropriate compartment as it passes along the wire.

Contd.....

1.5. Method of Working.

To enable the machine to handle a problem, it is first necessary to compile the data and the programme of orders in a suitable form. Thus if the machine is to calculate the wages to be paid to an employee, details of the number of hours worked, the rate of pay, any bonus payment to be made, and other details required in the calculations must be assembled; and a complete programme of orders laying down all the steps required to produce the desired results must be formulated.

When the start button is pressed, the machine then automatically carries out the following steps:-

- (a) it reads and stores the programme of orders, which now indicate all further steps to be carried out;
- (b) it reads and stores the data already assembled;
- (c) it carries out the calculations indicated in the programme, to arrive at the Base Pay, Gross Wages, Tax Deduction and Net Pay;
- (d) as each required result is produced, it holds it in a specific position in the store;
- (e) it reads the results from these positions and either prints them in a pre-arranged fashion which is governed by the programme of orders, or it records them in code in some way more suitable for future use.

Thus when once the machine has been started, it will automatically follow the orders until the operations have been completed, without outside intervention. For this reason the machine is called an "automatic" calculator.

A machine capable of carrying out these functions at all is remarkable, but what is even more remarkable is the speed at which they are performed. The addition of two numbers can be carried out two-hundred and fifty times a second, and multiplication one hundred and twenty times a second. But the high effective speed of the machine is achieved not solely by the speed of each calculation, but also because a long and complicated series of calculations can be carried out without pause.

When necessary the machine can even choose between two alternative courses of action, provided that the alternatives have been foreseen, and the conditions governing the choice have been set down in the programme.

Contd.....

1.6. Application to Clerical jobs.

A machine of this type can, it is believed, be made to perform a great number of clerical jobs, particularly those involving arithmetic. The structure of the machine is naturally much more complex than existing office machines and the cost of building one correspondingly higher. Nevertheless the speed of operation and the possibility of doing the whole clerical job from the taking in of the original data to the printing of the final result make its potentialities very great.

In order to use the machine it will be necessary for each clerical job to be analysed and expressed as a programme that the machine can accept and interpret. Before considering how these programmes are prepared for any clerical job it is necessary to consider how the machine is constructed and the form in which it holds and uses the information given to it.

Contd....

2. General Organisation of the Machine.

2.1. Essential Stages in the process by which the Machine handles a problem.

In order that a machine may handle a problem there are four main stages which must be completed:-

- (a) the machine must be started to enable it to carry out the subsequent steps;
- (b) the programme of orders and the data for the problem must be taken into the machine;
- (c) the required series of calculations must be carried out by following the orders which have already been taken in;
- (d) the answer or answers to the problem must be presented in either printed or coded form.

It is proposed here to describe the main organs that make up the machine and the role they play in carrying through these stages.

2.2. Main Parts of the Machine.

- (a) Co-ordinator
- (b) Store
- (c) Starter
- (d) Reader
- (e) Computer
- (f) Recorder

The rough relationship of these parts is represented by the schematic diagram attached to page 9.

2.3. (a) The Co-ordinator is the "nerve centre" of the machine because, with one exception, it gives the stimuli to the other parts to carry out their functions, and receives back messages when the operations have been completed so that it can proceed to initiate the next step. The dotted lines in the diagram indicate the links with the other parts by which control is exercised. The full lines show the path followed by pulses representing information flowing in the machine, with arrow heads to show the direction of flow.

Information coming from any of the other parts of the machine always passes into the same Main Discharge Line which leads into the Co-ordinator; from the latter the information passes along the Main Feed Line which has connections with the Store, the Computer, and the Output Mechanism.

The course of action of the Co-ordinator itself is governed by the programme of orders. The pulses constituting an order are interpreted by the Co-ordinator which then sends out the appropriate stimuli to the other parts of the machine.

Contd.....

2.4. (b) The Store is the part of the machine in which are held at various times during the calculations:-

- (i) all orders to be used by the Co-ordinator;
- (ii) the data for the problem;
- (iii) all intermediate results of calculations;
- (iv) the final results required.

2.5. (c) The Starter. Since the Co-ordinator works by "reading" orders which are held in the store, it will be seen that until sufficient orders have been stored to enable the programme and data for the required problem to be accepted, the Co-ordinator cannot start working. At the same time the programme and data cannot be taken into the store until the Co-ordinator gives the necessary stimuli.

It is the task of the Starter to supply the preliminary orders to the Store and then give the necessary stimulus to the Co-ordinator so that it reads what has been put in the Store and commences its task. This stimulus given by the Starter to the Co-ordinator is the one case referred to above where the stimulus is not provided by the Co-ordinator.

When the Starting-button of the machine is pressed, the Starter supplies to the Store a pre-arranged sequence of orders which are permanently wired into the machine and at this stage are converted into pulse groups. These preliminary Orders are the same for all problems and, by following them, the Co-ordinator can ensure that information required for the job in hand is taken into the machine.

2.6. (d) The Reader is the means whereby the data and the programme of orders for a problem are taken into the machine. It is a device which can "read" information presented to it in the form of holes punched in a paper tape, and convert them into groups of pulses to be put into the Store.

When the Preliminary Orders have entered the Store, the Co-ordinator reads and acts upon each in turn. The effect of these Preliminary Orders is to cause the information punched in the paper tape to be taken in through the Input Mechanism piece by piece, and presented to the Co-ordinator. This information is placed by the Co-ordinator in appropriate compartments of the Store determined by the Starting Orders so that it can be found when required. In this way the programme of orders and the data for the problem can be taken in, and when they have been accepted the Co-ordinator can proceed to the first order of the programme.

2.7. (e) The Computer is that part of the machine which performs the calculations by means of electronic circuits. It is built to carry out two primary types of calculation, addition and multiplication. Other types such as subtraction, division, etc. are carried out by using the circuits for the primary calculations together with other auxiliary circuits.

When the Co-ordinator "reads" an order to make a calculation it interprets it and causes the transfer of the required data from the Store to the Computer, where it is held in "registers"; the Co-ordinator then provides the appropriate stimulus to the Computer to carry out the desired calculation on the data.

Contd.....

The Computer contains three "registers" which are, in fact, Delay Storage circuits similar to those used for the Store (see 1.4) but of shorter length since they are required to hold only a single number at a time. These registers are used to provide temporary storage, in readily accessible form, for numbers on which arithmetical operations are being carried out.

The most important of these registers is the Accumulator, which is connected to circuits which ensure that any number presented to it is automatically added to the contents of this register. The result of a series of calculations can thus be held here. If the Accumulator is needed for a calculation not involving the number actually held but which is, however, required at a later stage in the programme, then the contents may be transferred to the Store by a special order inserted at this point in the programme. Similarly the Accumulator can at any time be cleared by inserting an order to that effect in the programme.

The other registers are the Multiplier and Multiplicand registers, which are used to hold the factors required for multiplication.

2.8. (f) The Recorder is the means whereby the results held in the Store are recorded in some more permanent form. When the Co-ordinator "reads" the appropriate order in the Store it sends the information to the Recorder, which may be either a printing device or a punching machine which perforates a tape with the information in coded form. In the latter case an auxiliary printing device may be used to decode and print the information later.

2.9. How the Parts work together in dealing with a Problem.

The way in which the parts of the machine work together can be illustrated by considering what happens when the machine is given the problem of adding two numbers "x" and "y".

The following is a somewhat simplified version of what happens. The programme, prepared for the problem beforehand, would be in the form of four orders:-

- | | |
|------------|---|
| Order No.1 | Add number x into the Accumulator of the Computer. |
| " No.2 | Add number y into the Accumulator of the Computer. |
| " No.3 | Transfer the contents of the Accumulator into a specified compartment of the Store. |
| " No.4 | Print the contents of that compartment of the Store. |

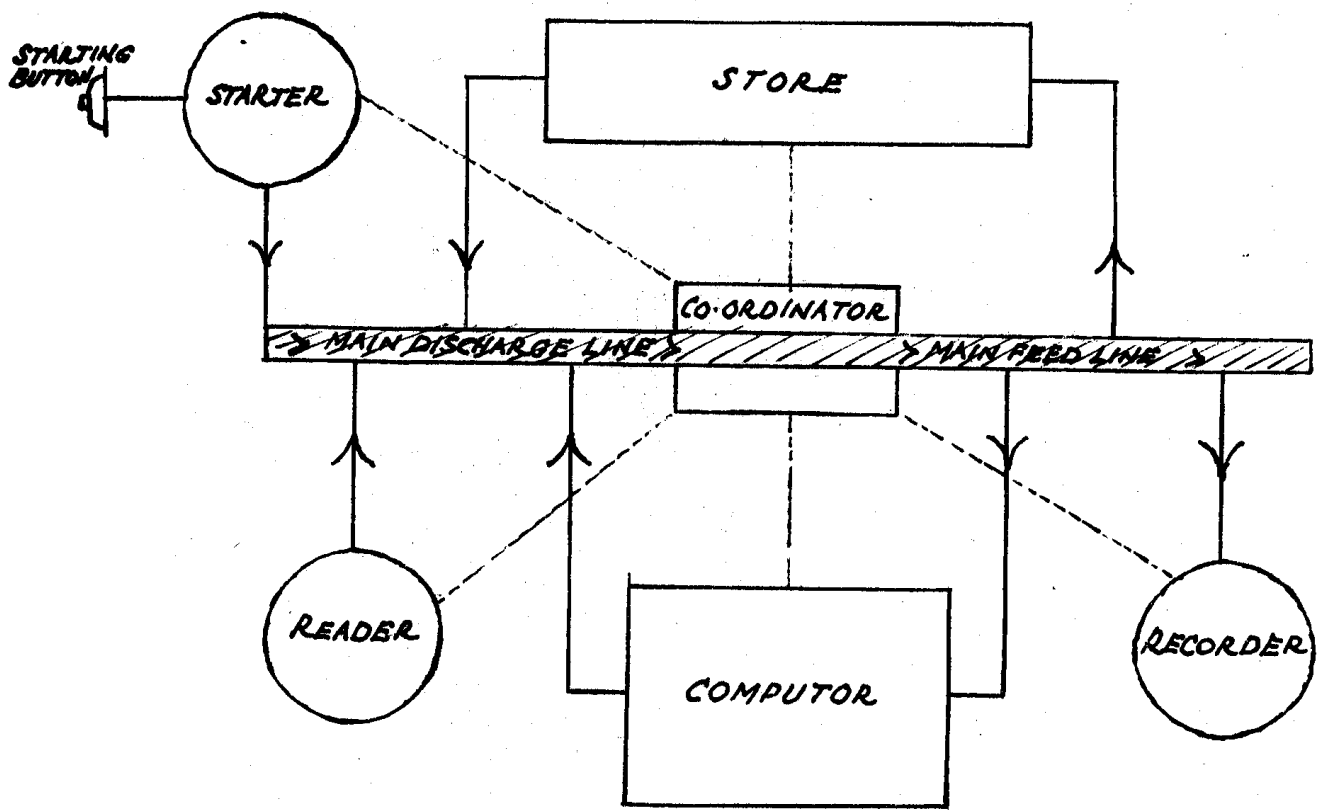
This programme and the numbers "x" and "y" would be punched on tape in coded form and attached to the Reader.

Contd.....

When the Starting-Button is pressed, the following processes take place:-

- (a) The Preliminary Orders held in the Starter are passed, in the form of pulses, through the Co-ordinator into the first compartments of the Store.
- (b) When all the Preliminary Orders have been so transferred, the Starter sends a stimulus to the Co-ordinator which then proceeds to carry out the Preliminary Orders. In accordance with these orders the Co-ordinator takes in through the Reader the four orders which form the programme, and the two numbers which form the data, and stores them in the next six compartments of the Store.
- (c) The last of these Preliminary Orders refers the Co-ordinator to Order No.1 of the programme, and the following sequence of events takes place:-
 - (i) Order No.1 is discharged into the Main Discharge Line;
 - (ii) The Co-ordinator interprets Order No.1;
 - (iii) The Co-ordinator causes the compartment of the Store containing "x" to discharge its contents into the Main Discharge Line;
 - (iv) The number "x" is fed into the Accumulator of the Computer;
 - (v) The Co-ordinator refers to Order No.2 and it is discharged into the Main Discharge Line;
 - (vi) The Co-ordinator interprets Order No.2;
 - (vii) The Co-ordinator causes the compartment of the Store containing "y" to discharge its contents into the Main Discharge Line;
 - (viii) The number "y" is fed into the Accumulator of the Computer and is automatically added to "x" to give " $x + y$ ";
 - (ix) The Co-ordinator refers to Order No.3 and it is discharged into the Main Discharge Line;
 - (x) The Co-ordinator interprets Order No.3;
 - (xi) The contents of the Accumulator of the Computer are discharged into the Main Discharge Line;
 - (xii) These contents are fed into the specified compartment in the Store;
 - (xiii) The Co-ordinator refers to Order No.4 and it is discharged into the Main Discharge Line;

Contd.....



- (xiv) The Co-ordinator interprets Order No.4.
- (xv) The Co-ordinator causes the compartment of the Store containing $(x + y)$ to discharge its contents into the Main Discharge Line;
- (xvi) This information is fed to the Recorder.
- (xvii) The Recorder prints this information.

This may seem a very involved process for the simple addition of two numbers. It should, however, be realised that all these steps are in fact necessary, in one form or another, whatever means is used to carry out the addition, even in the case when a human computer does it in his head.

2.10. Physical Characteristics.

Because the organisation of the machine has been described in terms of the six main parts given above, it must not be thought that the physical lay-out of the machine corresponds closely to them. Physically the machine can be divided under three headings:-

- (a) the reader and recorder, with their associated apparatus;
- (b) the storage tubes;
- (c) the electronic circuits.

The reader, recorder and associated equipment are pieces of standard teleprinter equipment which have been slightly modified:-

- (i) A Keyboard Punch, with a keyboard similar to a typewriter, is used to produce the perforated tape which expresses in code form the information to be fed into the machine. This instrument is operated manually.
- (ii) The Reader itself senses this information by means of "feelers" which detect the holes in the tape and produce a corresponding pattern of electrical pulses.
- (iii) The Recorder may be an Automatic Punch, operated by pulses from inside the machine instead of by a keyboard. This form of Recorder is used when the results of calculations are required in the form of punched tape.
- (iv) Where the machine is to produce a printed record, a Teleprinter is used as the Recorder, which is actuated by pulses from the machine and prints the results in much the same way as a typewriter.

Contd.....

The Delay Storage tubes are arranged in two batteries of 16 Tubes each. Each battery is enclosed in a long box about 6' x 2' x 2' so as to avoid sudden temperature changes. The tubes are each about 6' long, of ordinary mild steel with a bore of 1".

By far the largest space is occupied by racks of electronic equipment, comprising valves of different kinds and associated components. The 13 racks of equipment are related in the main to the six parts that have been described, as follows:-

- 4 are associated with the Store;
- 4 with the Main Control.
- 3 with the Computer.
- 1 with the Input and Output Mechanisms;
- 1 with the supply of power and pulses.

Each rack is 7' high, 2' 6" wide, and about 1' deep and accommodates about 10 to 12 panels in which are set the valves and associated components. The total area required to accommodate the whole machine is about 20' x 10'.

In all there are about 3,000 valves which consume a total of about 10 kilowatts, i.e. the same as five two-bar electric fires.

3. Forms in which information appears in the machine.

The information which the machine may have to use is of three kinds:-

- (a) numbers;
- (b) orders to be executed;
- (c) words which it is desired to print with numbers in the results.

3.1 Numbers.

In order to understand how the Machine deals with numbers we must first consider various ways of expressing numbers. Consider first the numbers "one" to "twenty" expressed in words:-

One	Five	Nine	Thirteen	Seventeen
Two	Six	Ten	Fourteen	Eighteen
Three	Seven	Eleven	Fifteen	Nineteen
Four	Eight	Twelve	Sixteen	Twenty

Each of these (and indeed every such number in words) expresses a unique quantity. To the farmer it may be a quantity of sheep; to the child a quantity of toy bricks; to the sparrow a quantity of eggs. It does not matter whether the individual can count or make calculations - reduced to its simplest terms the number is the conception of a quantity of things in the mind of that individual.

The Romans expressed these twenty numbers as:-

I	V	IX	XIII	XVII
II	VI	X	XIV	XVIII
III	VII	XI	XV	XIX
IV	VIII	XII	XVI	XX

But this system of expression is very inconvenient for making calculations as will be seen if this is attempted. It was not until the system we are familiar with was developed that calculations could conveniently be made. This is called the "Arabic" notation and is based on the use of "digits" i.e. the fingers and thumbs on a person's two hands. Counting up from one to nine is denoted by the nine numerals 1, 2, 3, -----9. Reaching ten there is a complete set of ten digits. The number of sets of ten digits is expressed in the same numerals 1, 2, -----9 but in a position one place to the left. The odd digits in addition to the sets of ten are shewn in the first position from the right. For exact sets of ten it is necessary to put a numeral in first position to denote there are no odd digits, so the numeral "0" is introduced, making ten numerals in all. Thus the numbers one to twenty are expressed:-

Contd.....

	<u>digits</u>		<u>Set of ten</u>	<u>digits</u>
Nought	0	Ten	1	0
One	1	Eleven	1	1
Two	2	Twelve	1	2
Three	3	Thirteen	1	3
Four	4	Fourteen	1	4
Five	5	Fifteen	1	5
Six	6	Sixteen	1	6
Seven	7	Seventeen	1	7
Eight	8	Eighteen	1	8
Nine	9	Nineteen	1	9

Reaching twenty we have 2 sets of ten. So we can continue up to 99, i.e. 9 sets of ten plus 9 digits.

For a hundred we have now "ten sets of ten digits". This we express by using a third column.

Similarly we need to use a fourth column for thousands, a fifth column for ten-thousands, etc.

So by sufficient columns we can express any number by means of the ten digits 0 to 9. When we put down 5,783 we mean:-

five thousands
 + seven hundreds
 + eight tens
 + three single digits

*Implication of
unnecessary
complexity*

or mathematically $(5 \times 10^3) + (7 \times 10^2) + (8 \times 10) + 3$. This is the Decimal System with which we are familiar and in which we are readily able to make calculations.

Now we can consider how the Machine is to express numbers. Unfortunately it is only possible to build a Machine which can distinguish the ten numerals 0, 1, 2, -----9, by introducing technical difficulties which it is desirable to avoid. In fact it is only convenient, as will be shown later, to distinguish two numerals and these inevitably are 0 and 1.

The Machine is, therefore, like a man with only one digit on each hand. It must perforce express all numbers in terms of two numerals 0 and 1. Reaching two, it has a "set of two digits" for which it uses one place to the left. Counting up to three is therefore denoted:-

	<u>digits</u>		<u>Set of two</u>	<u>digits</u>
Nought	0	Two	1	0
One	1	Three	1	1

Contd.....

For "two sets of two" we need another column i.e. representing four, which suffices till we reach eight for which a fourth column is needed. Similarly additional columns are needed for sixteen, thirty-two, sixty-four, one-hundred and twenty-eight, and so on for all powers of two.

Thus instead of expressing numbers on the Decimal System in terms of 10 , 10^2 , 10^3 , etc. the Machine has to express them in terms of 2 , 2^2 , 2^3 , etc. which is called the Binary System. The number 11011 therefore means:-

One	x sixteen
+ One	x eight
+ Nought	x four
+ One	x two
+ One digit	

$$\text{i.e. } 16 + 8 + 2 + 1 = 27.$$

It will be clear from what has been said that just as any number can be expressed in decimal form in one, and only one, way, so it can be expressed by one binary number. It is thus possible to express any decimal number as its equivalent binary number. Since in ordinary life only the decimal system is used, it is desirable to examine the methods by which a number may be converted from decimal to binary form.

Thus the decimal number 37 will in binary form be a number composed of a row of 0's and 1's, and each position indicates whether the particular power of 2, corresponding to the number of places from the right, occurs in the number. Now 37 may be expressed as a number of groups of "two", with or without a final one:-

$$37 = (18 \times 2) + 1$$

Thus in the final position of the binary number there is a 1. The number (18) of groups of two can itself, by dividing by two, be expressed as a number of groups of "four", with or without an odd group of two:-

$$(18 \times 2) = (9 \times 4) + (0 \times 2)$$

So the number of 37 may also be written as:

$$(9 \text{ groups of } 4) + (0 \text{ groups of } 2) + 1.$$

and the last two binary positions are therefore 01. By dividing the number of groups of four (9) by 2, we can now express it as a number of groups of "eight", with or without a single group of four:-

$$(9 \times 4) = (4 \text{ groups of } 8) + (1 \text{ group of } 4)$$

and the last three binary positions therefore are 101.

Contd.....

By a further division, the 4 groups of 8 are:-

(2 groups of 16) + (0 groups of 8) which becomes
(1 group of 32) + (0 groups of 8) + (0 groups of 4).

Thus the whole number is:-

100101

i.e.

1 x 32	=	32
+ 0 x 16		
+ 0 x 8		
+ 1 x 4	=	4
+ 0 x 2		
+ 1 x 1	=	<u>1</u>
		37
		<u> </u>

Thus the binary equivalent of a decimal number may be obtained by repeated division by 2, the remainders at successive stages giving numerals in the binary positions from right to left. Eventually the quotient becomes 0, so that all other positions to the left are filled by 0; as they have no significance they are normally not written, as in the decimal notation.

The complete conversion of 37 to binary form is:

2	37	Successive	
2	18	<u>Remainders</u>	
2	9	1	first position from right
2	4	0	
2	2	1	
2	1	0	
2	0	0	
	0	1	last significant position
		0	

The Binary equivalent is therefore:-

100101

= 32 + 4 + 1 = 37.

Alternatively, the conversion of 37 to binary form could be carried out by a series of subtractions.

The positions of a binary number represent powers of two:-

1	16
2	32
4	64
8	etc.

Contd.....

The number 37 must be represented by the sum of a number of these powers, and it is clear by inspection that 32 is the highest of these.

$$\begin{array}{r} \text{Thus} \quad 37 \\ \quad \quad - 32 \\ \hline \text{Remainder} \quad 5 \end{array}$$

The most significant position of the binary number is therefore that representing 32 and is filled by a "1".

Since 16 and 8 cannot be subtracted from the remainder 5, their positions are filled by a "0".

The next position represents 4 which can be subtracted from 5 and so this is filled by a "1".

$$\begin{array}{r} \text{Thus} \quad 5 \\ \quad \quad - 4 \\ \hline \text{Remainder} \quad 1 \end{array}$$

2 cannot be subtracted from the new remainder so that its position is filled by a "0".

The next position represents "one" which can be subtracted so it is filled by a "1".

$$\begin{array}{r} \text{Thus} \quad 1 \\ \quad \quad - 1 \\ \hline \text{Remainder} \quad 0 \end{array}$$

The full number is therefore 100101.

The full process can therefore be shown as:-

$$\begin{array}{r} \text{minus } 1 \times 32 \quad = \quad \begin{array}{r} 37 \\ \underline{32} \\ 5 \end{array} \dots\dots\dots 1 \\ \text{minus } 0 \times 16 \quad = \quad \begin{array}{r} 5 \\ \underline{0} \\ 5 \end{array} \dots\dots\dots 0 \\ \text{minus } 0 \times 8 \quad = \quad \begin{array}{r} 5 \\ \underline{0} \\ 5 \end{array} \dots\dots\dots 0 \\ \text{minus } 0 \times 4 \quad = \quad \begin{array}{r} 5 \\ \underline{4} \\ 1 \end{array} \dots\dots\dots 1 \\ \text{minus } 0 \times 2 \quad = \quad \begin{array}{r} 1 \\ \underline{0} \\ 1 \end{array} \dots\dots\dots 0 \\ \text{minus } 1 \times 1 \quad = \quad \begin{array}{r} 1 \\ \underline{1} \\ 0 \end{array} \dots\dots\dots 1 \end{array}$$

Contd.....

3.2 Orders.

Provision has been made in the machine so that it is possible to order it to carry out up to 30 separate and distinct operations. The orders for these operations to be carried out are coded in the machine in the form of a binary number of 5 positions; that is, one binary number is assigned to each kind of operation. Thus the following operations, which are among those which can be performed by the machine, are specified by the binary numbers shown against them:-

- (a) Add into the Accumulator the number in a compartment of the Store to be specified: 11100
- (b) Subtract from the Accumulator the number in a compartment of the Store to be specified: 01100
- (c) Clear the Accumulator by transferring its total to a compartment of the Store to be specified: 00101
- (d) Copy the sub-total of the Accumulator into a compartment of the Store to be specified: 00111
- (e) Copy into the Multiplier Register the number in a compartment of the Store to be specified: 10101
- (f) Multiply by the contents of the Multiplier Register the number in a compartment of the Store to be specified, and add the product into the Accumulator 11111
- (g) Multiply by the contents of the Multiplier Register the number in a compartment of the Store to be specified, and subtract the product from the Accumulator. 10110
- (h) Read the next row of holes in the Input Tape and copy the information in a compartment of the Store to be specified. 01000
- (i) Record on the Output Medium the information held in the first 5 positions of a compartment of the Store to be specified. 01001

Contd.....

It will be seen that these functions all affect particular compartments of the Store; either the number to be used is held in a given compartment, or the result of the calculation is to be transferred to a given compartment. This is so for most of the functions carried out by the Machine, and thus an order must normally specify not only the function to be carried out, but the compartment of the Store concerned.

Each compartment is denoted by a number, as will be explained later in this section, and the appropriate number, in binary form, forms part of the order. A complete order is therefore composed of its function in the form of a binary number and its "address" in the form of a second binary number. The function part of the order occupies the last five positions in the compartment of the Store allotted to it, and the address the remaining positions.

3.3 Letters

In some problems it may be desirable to incorporate in the printed answer information involving letters as well as the numbers forming the answer. This is done by using a binary code for the letters of the alphabet. It so happens that teleprinters used for transmitting telegraphic messages employ a binary code for the letters. A similar code is used in E.D.S.A.C.

The code is:-

A	11100	H	10101	O	01001	V	11111
B	11101	I	01000	P	00000	W	00010
C	11110	J	01010	Q	00001	X	11010
D	10011	K	01110	R	00100	Y	00110
E	00011	L	11001	S	01100	Z	01101
F	10001	M	10111	T	00101		
G	11011	N	10110	U	00111		

3.4 Distinction between Orders, Letters and Numbers

It will now be realised that numbers, orders, and letters are all expressed in the Machine in the same way, i.e. in binary form. It follows that the same group of pulses in the Machine could represent a number, an order, or a letter. For instance, the number 28 is represented by 11100; so is the part of an order for the function of Adding; and so is also the letter A. For this reason when a programme is written, the function part of an order is indicated by letters.

Thus: Subtract (01100) is written "S"

Transfer to Store (00101) is written "T"

Contd.....

Since the Co-ordinator works according to the arrangement of pulses it receives and has no means of determining whether the pulses were intended as a number, an order or a letter, it is necessary to store the information in properly designated compartments of the Store and arrange the programme so that numbers, orders and letters will, when taken out of the Store, be treated appropriately. This is achieved by so arranging the orders in sequence that these compartments of the Store containing pulses which the Co-ordinator will interpret as orders are, in fact, orders and occur in the sequence in which they must be carried out; and that these compartments containing digits to be used as numbers contain the appropriate numbers for the problem.

The orders all follow one another in sequence in compartments of the Store, the numbers being called forth as they are required. Where a letter held in the Store is to be fed to the Recorder for printing, the programme must ensure that a signal is first fed to the Recorder which causes it to treat the following digits as a letter.

3.5 Punched Tape

We can now consider how binary information can be expressed in forms suitable for feeding into the machine, and for use inside it.

The first method is the use of punched paper tape, which can readily be produced by a Keyboard Perforator, which is used in conjunction with teleprinter equipment.

When a message is to be transmitted by teleprinter, the typist touches the keys as for typing, and a series of pulse groups, corresponding to the code given above is transmitted along a land line to the receiving instrument which is caused by the pulses to reproduce the message in print. The keyboard punch is somewhat similar to the teleprinter, but instead of transmitting pulses, it punches on a tape groups of holes whose pattern corresponds to the information.

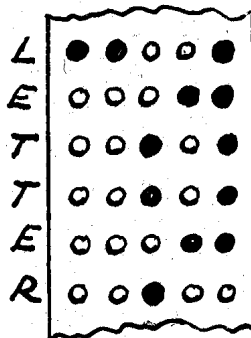
The tape provides five columns for possible holes, and as a key is pressed for a character, a row of holes is produced corresponding to the binary code for that character. A hole punched represents a "1" and an unpunched space an "0". Thus for the character E, for which the binary code is 00011, the tape would be punched:-



i.e. with holes punched in the two right hand columns only.

Contd.....

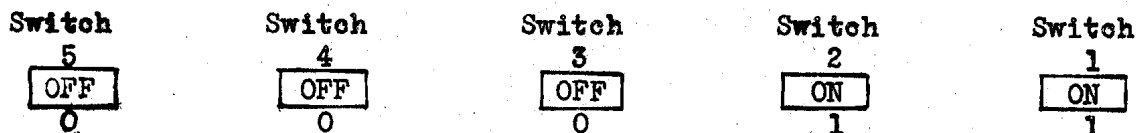
For the word LETTER the punched tape would appear:-



To put information into the Machine, punched tape is first produced in this way whether the information consists of numbers, orders, or letters. Numbers, orders and words are typed into the Punch just as into a typewriter.

3.6 Representation of numbers by Electro-magnetic Relay Switches

A "relay" is an electrically-operated switch which may be set at either "on" or "off". These two positions can be used to represent respectively 1 and 0, so that if a bank of these switches is used, they may be set so that their pattern of switching represents any binary number, one switch being used for each position. Thus the binary code for "E" (00011) can be represented by a bank of five switches as follows:



Only one binary number can, of course, be expressed in this way at a time.

In the machine five such switches are used in the Reader. Five feelers sense the holes for each line of tape in turn, and the switch corresponding to each column in which a hole is punched is set at "on"; the other switches remain at "off". In this way the binary number held on the tape as a pattern of holes is now converted into a pattern held as a setting of switches associated with electronic circuits.

The switches can now be used to control electric currents flowing in these circuits, as will be seen below, and the binary pattern is thus passed on, to be used inside the machine. The switches are then free to accept the pattern conveyed by the next row of holes.

The bank could, of course, include more than five switches, so that numbers of more than five binary digits could be held in this way.

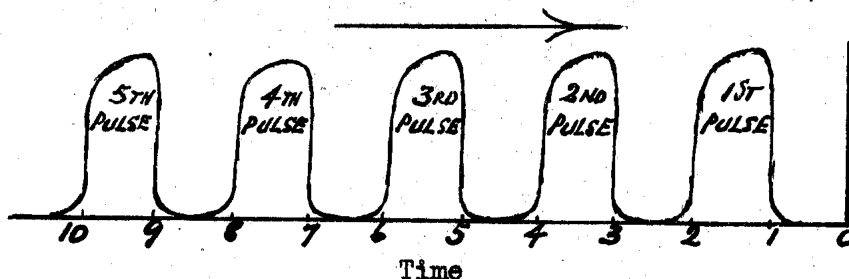
Contd.....

3.7. Representation of numbers by Trains of Electrical Pulses.

Numbers and other information move about in the machine in the form of trains of electrical pulses. To understand their nature imagine an electrical circuit with a switch in it which is at the "off" position. If it is switched on momentarily a "surge" of electricity passes from one end of the wire to the other. This switch could be turned on and off repeatedly and create a series of these surges, i.e. a train of electrical pulses through the wire.

In the machine there is a means of creating a continuous supply of such pulses. All pulses last the same length of time and the interval between them is also the same. Both the length of pulse and the interval are about one-millionth of a second (a micro-second), so that pulses are formed at the rate of one every two micro-seconds.

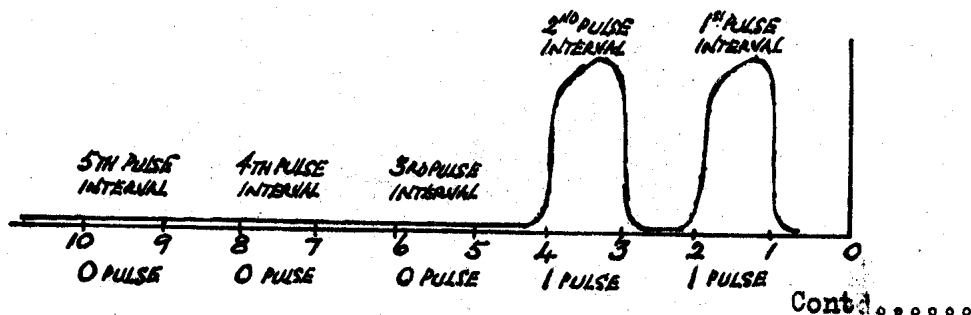
The following graph gives an illustration of the pulses passing a given point in the wire; the strength of the surges is represented vertically, and the passage of time horizontally:-



If now the pattern of these pulses is made to correspond to the "1"s and "0"s of a binary number, that number can be used in the machine in the form of a pulse train.

To effect this a pulse is fed in turn to each of the five electro-magnetic relays which have already been set up according to the binary pattern punched in the tape. Thus if (as in the example already given) the letter "E" (00011) has been accepted by these switches, the first pulse will be fed to switch 1, which is "On", and will be allowed to pass. The second, similarly, will pass through, but the third, fourth and fifth pulses will be suppressed since the corresponding switches are in the "off" position.

Thus the five pulses become:-



This is again the binary form of E; a pulse represents "1", and the absence of a pulse in a position where a pulse may occur represents "0".

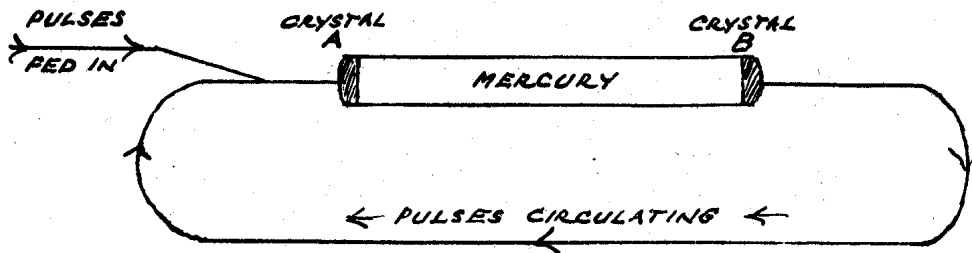
Binary numbers of any size can be expressed by using sufficient pulses in one train.

3.8. Pulse Vibrations in Mercury.

As has been explained before, the speed of electrical pulses along a wire is too great to allow their storage in that form. For this reason the principle of Delay Storage is used, whereby the electrical pulses are converted into mechanical vibrations in mercury.

To transform the electrical pulses into mechanical vibrations quartz crystals are used. When an electrical pulse as used in the machine is applied to a crystal it vibrates, and conversely, when the mechanical vibration is applied to a crystal, an electrical pulse is produced. The vibrations readily pass from the crystal into mercury in direct contact with one side of the crystal.

In the machine columns of mercury are held in metal tubes with quartz crystals at either end. Each tube is connected to an electrical circuit which sends the output pulses from B back into the mercury again at A. Pulses from outside the circulating system can also be fed in at A.



When a circulating pulse arrives at crystal A, the crystal sets up a vibration in the mercury which travels to crystal B. At crystal B this vibration sets up an electrical pulse which travels along the wire to crystal A again, and so on.

Thus if a series of pulses is circulating, its pattern (of pulses and gaps) will be faithfully reproduced in the mercury in the form of mechanical vibrations, and will again appear as a similar electrical pulse pattern after leaving the mercury.

The information represented by the pattern will not be affected in any way by the insertion of the Delay Line, but the time of circulation will be greatly increased because of the slower speed of the mechanical vibrations.

As has been mentioned in Section 1, it is because the speed of pulses in mercury is relatively slow compared with their speed in a conductor that it is possible to store pulse trains of some length.

Contd.....

The length of the pulse train in a single tube has been fixed so that it will store sixteen numbers, each of a size corresponding to a decimal number with ten positions. The number of binary positions corresponding to ten decimal positions is about 34; actually 36 pulse positions are provided. We therefore require 16 groups of 36 pulses i.e. 576 pulses. These are produced at the rate of about one every two micro-seconds (actually at 514,000 to the second), so that the 576 pulses will enter the tube in a period of $\frac{576}{514,000}$ seconds.

But if the pulse train is to be continuous the first pulse must be leaving the tube immediately after the last one enters, so that the length of the tube must be such that the first pulse has travelled the length during the time the 576 pulses have entered. Now a pulse travels through the mercury at the rate of 4,760 feet per second, so the number of feet travelled in $\frac{576}{514,000}$ seconds is $\frac{576 \times 4,760}{514,000} = 5'4''$ which is the length of tube required.

The time taken for a pulse to pass once round the storage tube circuit is termed a "major cycle". A major cycle lasts

$\frac{576}{514,000}$ seconds, i.e. 1.12 milli-seconds (thousandths of a second).

This is 576 times the interval between successive pulses of just under 2 micro-seconds (millionths of a second).

The time between the passage of the first pulse of one group of 36 pulses and the first pulse of the next group is called a "minor cycle", and is 70 micro-seconds. 16 minor cycles make a major cycle. The minor cycle is sub-divided into two compartments, each capable of holding either a number of half the size of that stored in a whole minor cycle, or an Order. (See 1.4.).

Since it is necessary to be able to identify the different compartments of the mercury tube, a means is provided for counting pulse intervals that have occurred from the starting up of the machine up to any point in time. The generator which produces the regular supply of pulses is called a "Clock Pulse Generator" because by its regularity it acts as a clock. The machine is also able to count the number of minor cycles which have elapsed since it was started, and in this way the Main Control is made aware of the moment at which a particular compartment of the store is leaving a particular memory tube. By this means the information in any compartment of the Store can be obtained as required at the moment it is leaving the tube.

Each of the 32 tubes is given a binary number to identify it; the numbers run from 00000 to 11111. Each of the 16 minor cycles in a tube is given another binary number, this time from 0000 to 1111 and the two compartments of a minor cycle are distinguished by 0 to 1. The full address of a compartment is therefore distinguished by a binary number with ten positions as follows:-

Tube	Minor Cycle	Half
00101	1100	1

The tube number is analogous to the number of a house in a street and the minor cycle number can be thought of as corresponding to the hour of the day at which a particular person would be available at that house. At any other hour the wanted person would be "not at home". If the time the person is at home is always the same every day of the week then a day is analogous to a major cycle because at regular intervals of a day we can always make contact with him. In order to extract information from a compartment we must know both the tube number and minor cycle number in which the information is contained, just as to make contact with the person we must know both the place and time of appointment.

4. Arithmetic with Binary Numbers.

In Section 3 it was explained how numbers can be expressed in binary form. The rules of Arithmetic in this form are similar to those for calculations with numbers in decimal form. Before we can consider how the machine can carry out such calculations we must first of all consider what these rules are. Some of the steps involved in making a calculation mentally are carried out sub-consciously as part of other steps. The machine, however, can only carry out steps that have been specifically laid down for it, so that each step must be considered separately and a set of simple rules formulated for it which do not require the exercise of judgement.

4.1. Counting.

The simplest arithmetical process is that of counting. This consists of the repeated addition of the number 1 to the previous answer, starting at 0. The process for decimal numbers is shown on the left hand side below -

0	
<u>1</u>	Starting with 0, 1 is added
1	to give a sub-total of 1
<u>1</u>	1 is again added
2	to give a new sub-total 2
.	And so on.
.	The rule so far is thus to take the next higher
.	numeral each time.
8	
<u>1</u>	
9	But when the highest numeral 9 is reached
1	this rule can no longer be applied.
<u>0</u>	The new rule is to revert to 0 in the first position
1.	and carry 1 to the second position.
<u>10</u>	
1	The previous rule then applies again for the
<u>11</u>	numeral in the first position until 9 is again reached.
.	
.	
19	
<u>1</u>	Now 0 again appears in the first position and 1 is
0	carried to the second position. The numeral in the
<u>1.</u>	second position is, therefore, increased by 1 each time
20	that the numeral in the first position reverts to 0.
.	
.	
.	
99	
<u>1</u>	Similarly when the second position reaches 9 it reverts
0	to 0 and 1 is carried to the third position and so on.
<u>1.</u>	
00	
<u>1.</u>	
100	

Contd.....

Now let us consider the rules for counting with binary numbers which for convenience we will take as having only five positions.

00000 First 1 in the first position is added into the total
00001 in the same position. The rule is therefore that 1
00001 added to 0 in any position gives 1 in the answer.

00001 Next 1 is added to 1 in the first position and
0 the answer reverts to 0 in this position while 1 is
0001. carried to the second position.
00010

00001 For the next addition 1 is added to 0 in the
00011 first position and 0 to 1 in the second position
00001 to give 1 in both positions of the answer.

00001 For the next addition 1 is added to 1 in the first position
0 giving 0 in the answer.
1. and 1 is carried to the second position. This 1 is added to
00 1 of the previous sub-total to give 0 in the answer,
1.. again carrying 1 to the third position
00100 where it is added to 0 to give 1 in the answer.

The rules are therefore:-

- (a) 0 in any position added to 0 in the sub-total gives 0 in the answer;
- (b) 1 in any position added to 0 in the sub-total gives 1 in the answer;
- (c) 0 in any position added to 1 in the sub-total gives 1 in the answer;
- (d) 1 in any position added to 1 in the sub-total gives 0 in the answer and 1 is carried to the next position where it is added to 0 or 1 in the previous sub-total according to the same rules.

4.2. Adding.

We must now consider the process of adding two numbers together. Whereas Counting consisted of the repeated addition of the same digit to a given number, we now have the single addition of two numbers each consisting of several digits.

Consider the two binary numbers:-

a. 01101100

and b. 00101010

They have been chosen because, as will be shown, the additions of digits in the various positions illustrate all the rules that apply. The addition must be performed by some method which does not involve the simultaneous addition of more than two digits since the construction of the machine is such that at any one time it cannot do more than add 1 and 1 together. The rules must therefore be devised on this basis.

Contd.....

The process of addition starts from the least significant position, i.e. the right hand position; this is obviously necessary if we are to carry from one position to the next.

Thus the first three digits are added together as follows:-

a100	In the first position there is 0 in both
b010	<u>a</u> and <u>b</u> , so 0 must appear in the total <u>t</u> .
t	<u>.....110</u>	In the second and third positions we have 1
		in one of the numbers, and 0 in the other,
		and in both cases 1 is required in <u>t</u> and
		there is no carry figure.

If we now take the next two digits:-

a01 100	In the fourth position we have 1 in both
b01 010	<u>a</u> and <u>b</u> and, as in counting, 0 is required
t ₁00 110	in the total, <u>t</u> ₁ , and 1 must be carried
c ₁10 000	to the fifth position. This 1, carried to
t ₂	<u>....10 110</u>	the next position, is shewn separately as a
		carry number, <u>c</u> ₁ .
		In the fifth position 0 occurs in both <u>a</u> and
		<u>b</u> , to give 0 in <u>t</u> ₁ , but we must now add the 1
		which has been carried to this position in <u>c</u> ₁ ,
		to obtain the total <u>t</u> ₂ ; since there is 0 in
		<u>t</u> ₁ , and 1 in <u>c</u> ₁ , 1 appears in <u>t</u> ₂ .

In the sixth, seventh, and eighth positions:-

a	011 01100	In the sixth position there is again 1 in
b	001 01010	both <u>a</u> and <u>b</u> , and we write 0 in <u>t</u> ₁ in that
t ₁	010 00110	position and carry 1 over to the next position
c ₁	010 10000	in <u>c</u> ₁ . <u>c</u> ₁ must next be added to <u>t</u> ₁ in this sixth
t ₂	000 10110	position but since there is 0 in both <u>t</u> ₁ and <u>c</u> ₁ ,
c ₂	100 00000	0 is required in the answer <u>t</u> ₂ .
t ₃	<u>100 10110</u>	In the seventh position there is 1 in <u>a</u> and 0 in
		<u>b</u> , which gives 1 in <u>t</u> ₁ . Now when we perform the
		second addition, of <u>t</u> ₁ and <u>c</u> ₁ , for this position,
		we have 1 in both numbers, and must write 0 in
		<u>t</u> ₂ and carry 1 to the next position. This "1"
		is shewn in a second carry figure which must
		again be added to the total <u>t</u> ₂ to give the
		final total <u>t</u> ₃ .

But this carry figure is in the eighth position which we must now consider.

There is 0 in a and b, and therefore in t₁. There is also 0 in c₁; so in adding t₁ and c₁, there is 0 in t₂. Finally in adding t₂ and c₂, the 1 in c₂ in this position produces 1 in t₃.

Contd.....

Condensed Form

a 01101100
 b 00101010
 t, 01000110
 c, 01010000
 c₂ 10000000
 t₂ 10010110

It will be clear that we could have avoided the extra addition of c_2 to t_2 by adding both c_1 and c_2 to t_1 at the same time. This would not be possible if 1 occurred in t_1 , c_1 & c_2 in the same position. Here, as we see, this is not so. But, in fact, there never can be a carry in c_1 and c_2 in the same position. This is so because the carry to c_2 only occurs when t_1 has a 1 in the previous position; and there is never a carry to the ^{next} position of c_1 when t_1 has a 1. For this reason c_1 and c_2 can always be added to t_1 simultaneously.

The example illustrates all the possible combinations of digits to be added in any position, viz: -

- (i) two 0's
- (ii) 0 and 1
- (iii) two 1's

The same rules apply whether the digits of the two original numbers are being added or those of the first total and the two carries.

The method of adding two numbers is, therefore, to take the positions in turn from right to left and to add each pair of digits in two stages; in the first stage for a given position the digit of a first total and of a first carry to the next position are obtained; in the second stage for that position the digit in the first total is added to the digits of both the first and second carries from the previous position.

The rules for adding the two digits for any position in the two stages are: -

First Stage.

- (a) a digit 0 in both numbers gives 0 in the first total and 0 in the following position of the first carry number.
- (b) a digit 1 in one of the numbers and 0 in the other gives 1 in the first total, and 0 in the next position of the first carry number.
- (c) a digit 1 in one of the numbers added to 1 in the other gives 0 in the first total and 1 in the following position of the first carry number.

Contd.....

Second Stage.

- (d) a digit 0 in both the first total and both the first and second carry numbers gives 0 in the answer.
- (e) a digit 1 in the first total and 0 in both the first and second carry numbers gives 1 in the answer.
- (f) a digit 1 in the first or second carry number and 0 in the first total gives 1 in the answer.
- (g) a digit 1 in the first total and 1 in either the first or second carry number gives 0 in the answer and 1 in the next position of the second carry number to be added by the same rules.
- (h) in any position there can never be 1 in both the first and second carry numbers.

4.3. Multiplication.

In multiplying two decimal numbers together the normal process is:-

254	Multiplicand
174	Multiplier
1016)	
1778.)	Partial
254..)	products
44196	

Here the multiplicand is multiplied by each digit of the multiplier in turn, but since each digit of the multiplier, according to its position, has a different decimal significance, the answer to each of these multiplications (i.e. each partial product) must be shifted successively 1 place to the left of the previous partial product. These partial products are then added together to give the answer.

In performing multiplication with binary numbers a similar process is followed, and an analogous set of rules are required.

Contd.....

Take the two numbers:-

a 10110 multiplicand

b 1011 multiplier

a 10110
 b ...1
 p₁ 10110
 t₁ 10110

For the first position of the multiplier from the right - the number a must be multiplied by 1. But multiplication by 1 is just a matter of putting down the multiplicand i.e. taking a as the first partial product, p₁. If we keep a running total then the first total to date, t₁, is also a.

a 10110
 a₁ 10110.
 b ...1.
 p₂ 10110.
 t₁ 10110
 t₂ 1000010

For the second position of the multiplier, we must ensure, in performing the multiplication, that the binary significance of this position is given effect. This is most easily done by immediately moving the multiplicand one position left, to give a₁. The second digit of the multiplier is 1, and so the second partial product p₂, is also a₁. p₂ must now be added to the previous total to date t₁, to give the new total to date t₂, by the rules for addition given in 4.2. above.

a₁ 10110.
 a₂ 10110..
 b ...0..
 p₃ 00000..
 t₂ 1000010
 t₃ 1000010

For the third position of the multiplier, a₁ must again be shifted one place to the left to give a₂, but since the multiplier digit is 0, the partial product p₃ here is 0, and the total to date, t₃, is therefore the same as t₂.

a₂ 10110..
 a₃ 10110...
 b ...1...
 p₄ 10110...
 t₃ 1000010
 t₄ 11110010

For the fourth position of the multiplier, which is 1, the multiplicand a₃ is shifted to give the next partial product p₄. This is added into the previous total to date t₃ to give the final product, t₄.

Thus the rules for binary multiplication may be expressed as:-

For each position of the multiplier in turn, starting from the right:-

1. If there is 1 in the multiplier add the multiplicand into the answer according to the rules given in 4.2.
2. If there is 0 in this position do not add the multiplicand.
3. Shift the multiplicand one place to the left.

Contd.....

4.4 Negative Numbers

The machine has been designed to deal with negative numbers as well as positive ones. A negative number is, of course, an ordinary number with a minus sign before it, e.g. -37, which is a number less than 0 to the extent of 37; when added to 40 it gives the answer 3.

One of the main reasons for designing the machine to deal with negative numbers is that by so doing it is possible for the machine to do subtraction without having to provide a special mechanism. If it is desired to subtract the number a from another number b this is the same thing as adding the negative number -a to b; this, as will be shown, the machine can do with the adding mechanism provided.

Apart from the process of subtracting, the application of negative numbers may be illustrated by considering a debit balance in a Supplier's Ledger Account. Normally one expects a supplier to be owed something, and therefore his account has a credit balance; if however he has been paid £1 more than was due, his account could be said to have a balance of "-£1".

Negative numbers can be worked upon in the binary system in just the same way as they can in the decimal system, e.g. "-37" in binary form becomes "-101101". To deal with these negative numbers the machine must be capable of accepting, storing, recognising and acting on them in accordance with the correct arithmetical principles. But, as has been explained, the machine has only two ways of expressing numbers, viz: a pulse of electricity or no pulse. The machine cannot therefore distinguish between -1 and +1 in a direct manner and some indirect means must be found.

The largest positive number the machine can accept is one of 34 binary positions (see Sec. 3.8) so that the largest positive number dealt with is 1111.....111111 with 34 positions in all. A number of this size occupies a minor cycle of 36 positions. It is therefore possible to allocate the 35th position in the minor cycle to represent a negative number. This negative number is -100 with 34 0's, i.e. ignoring the sign it is one greater than the largest positive number. It is the only purely negative number that can be expressed in the machine and for convenience we have termed it the "Negative Digit". The Negative Digit can be used in conjunction with positive numbers to represent any negative number, for if we want to store the negative number -10011, we first put in the machine:-

The Negative Digit 100.....00000000 (35 positions)
which is bigger than the number 100101
to the extent of 111111011011 (34 positions)

If therefore we put into the machine together this latter positive number and the Negative Digit we have the negative number required. It appears in the machine as one number, viz:-

+
1 11.....11011011 with 35 positions.

The positive part of the number is called the "complement" of the negative number which is being expressed and it is, therefore, the latter deducted from the Negative Digit.

Contd.

In order to put a negative number in the machine it is necessary first to find the complement of the number and then to combine it with the Negative Digit. The complement of a number can be arrived at as follows:-

First put a "1" in every position where there is a "0" and "0" in every position where there is a "1", so as to give what we may call "its Reflected Number"; then add 1 to obtain its complement, thus:-

Number	<u>00100101</u>
Reflected Number	11011010
Add 1	<u>1</u>
Complement	<u>11011011</u>

This must always give the complement, for the Reflected Number is the result of deducting the original number from 1111.....111, which is 1 less than the Negative Digit 1000000; so the Complement, which is the result of deducting a number from the Negative Digit, must be 1 more than the Reflected Number.

The adding of 1 to the Reflected Number produces 1 in the first position i.e. the same as in the original number. Had the digit in the first position been 0 in the original number, there would have been a 1 in this position of the Reflected Number, and the adding of 1 would produce 0 again in this position of the complement, i.e. the same as in the original number. The adding of 1 would also have carried 1 to the next position and so produced the same digit in the second position of the Complement as in that position of the original number whether that digit was 1 or 0 -

	•••••	or
Original Number	<u>10110</u>	<u>10100</u>
Reflected Number	01001	01011
Add	<u>1</u>	<u>1</u>
Complement	<u>01010</u>	<u>01100</u>

Again if there was 0 in the second position of the original number, the adding of 1 to the digit in the Reflected Number produces a "carry 1" to the third position; and this has the effect of making the digit in the third position of the Complement the same as that of the original number. This process of restoring the original digits goes on up to and including the earliest position of the original number that has 1.

Contd.....

The rules for obtaining the complement of a number are:-

- (a) starting from the right, put in the complement the same digit as in the corresponding position of the original number, up to and including the first position of the number that has 1;
- (b) for subsequent positions to the left put the opposite digits in the corresponding positions.

A negative number which will be fed to the machine as a positive number with a negative symbol must be expressed in the machine as 1 in the 35th place and the complement of the number in the first 34 places. Therefore the programme of orders on which the machine works must be so planned that, when the negative symbol is detected by the machine, it automatically converts the number into this proper form.

Contd. . .

4.5. Addition of Negative Numbers.

NB. For convenience the 11th to 34th binary places of each number have been omitted so that in the illustrations that follow the 11th place must be interpreted as the 35th; it is the only negative number available viz: -1000000000.

Consider the addition of +37 to 68. In the machine 68 takes the form 1000100 and -37 the form 1 1111011011 as above. The rules for this addition must clearly be the same as for positive numbers, as the machine recognises numbers as pulse-trains only and must apply the rules laid down to positive and negative numbers alike. Applying the rules for positive numbers (4.2.) to this addition we have:-

	- +	
68	0 0001000100	
<u>- 37</u>	<u>1 1111011011</u>	
First total	1 1110011111	
First carry	0 001.....	
Second carry	<u>1 11.....</u>	
<u>31</u>	<u>0 0000011111</u>	
	= 16 + 8 + 4 + 2 + 1	
	= 31	

It will be seen that the rules still apply satisfactorily; the Negative Digit is cancelled out by the eleventh digit of the second carry number which, since it arises from the positive digits of the tenth position, is itself positive.

But if the positive number is less than the negative number to which it is added the Negative Digit would not be cancelled, thus:-

	- +	
28	0 0000011100	
<u>- 37</u>	<u>1 1111011011</u>	
First total	1 1111000111	
First carry11....	
Second carry	
<u>- 9</u>	<u>1 111110111</u>	
	= minus 000001001	
	= minus (8 + 1)	
	= minus 9	

Contd.....

Similarly for the addition of two negative numbers the Negative Digit appears in the answer:-

- 37	- +	1 1111011011
<u>- 9</u>	First total	1 1111110111
	First carry	0 0000101100
	Second carry	11 111.1..11.
<u>- 46</u>	Answer1.11...
		*1 1111010010
		= minus 0000101110
		= minus (32 + 8 + 4 + 2)
		= minus 46

* Note. The 1 in the position to the left of the Negative Digit is not treated by the machine as a number but is ignored.

4.6. Subtraction.

It will be seen that this process of adding a negative number is in fact equivalent to subtracting a positive number. Thus, provided that the machine is able to convert a positive number into a negative number, subtraction can be carried out by obtaining the complement of the number and attaching to it the Negative Digit, and then using the adding circuit in the normal way.

The means by which the machine converts a positive number into a negative number is described in Section 5.

4.7. Multiplication of Negative Numbers.

Sometimes it may be necessary to multiply negative numbers, as, for instance, in Sales Invoicing when a return has to be credited.

Consider the multiplication of 37 by -5.

37	is	100101
		- +
-5	is	1 111011 (ignoring the places from 7 to 34).

Each of these numbers has six positive binary digits, so that the product will require 12 positive digits, the 13th becoming the position for the "-1" of negative numbers.

Contd....

This multiplication is carried out in the same way as in multiplying positive numbers; each digit of the multiplier is taken in turn, the multiplicand is shifted and where there is a "1" in the multiplier the multiplicand is added in. When the digit representing -1 in the multiplier is reached, however, the amount to be added in will be $-1 \times 100101 = 100101$ and this must be converted into a negative number which can be handled by the machine. Thus -100101 is replaced by its complement, preceded by the Negative Digit.

The multiplication is thus in full:-

		- +		100101	
	37 Multiplicand			111011	
	<u>-5</u> Multiplier			-----	
	$p_1 = t_1$	- +		100101	
	p_2			100101	
	t_1			1101111	
	p_3			100101	
	t_2			110010111	
	p_4			100101	
	t_3			1111100111	
	p_5			100101	
	t_4			100010000111	
	p_6		1 011011		Complement of) multiplicand)

	<u>- 185</u>		t	1 111101000111	Total product

	= minus			000010111001	
	= minus			(128 + 32 + 16 + 8 + 1)	
	= minus			- 185	

It will be seen, therefore, that the machine when obtaining the partial product for the Negative Digit in the multiplier correctly arrives at the negative equivalent of the multiplicand. This negative partial product is then added to the accumulated total of the other partial products to give the correct negative result.

Next we have to consider a negative multiplicand and a positive multiplier, as may arise in Sales Invoicing when a rebate is to be allowed on goods that have been sold. Thus in multiplying -37 by 5, if we again use 6 positive positions for each of these factors, the multiplicand is 1 011011, and the multiplier is 000101. But we will require 12 positive positions in the answer, and the 13th will be taken as the Negative Digit.

Contd.....

Therefore when we add a partial product into the answer, its first 12 positions have got to be positive positions. So when, as here, we have a negative multiplicand (i.e. its seventh position is the Negative Digit), it is necessary so to convert the partial product, without altering its value, that the first 12 positions are positive and the 13th is the Negative Digit.

The multiplicand here is 1 011011, the positive part of which is the complement of 100101. But 100101 (with 6 digits) is the same as 000000100101 (with 12 digits) and therefore if we take the complement of this 12 digit number and attach to it the Negative Digit in the 13th position, we shall again have the original multiplicand but with the appropriate number of digits for adding into the total:-

$$\begin{array}{r}
 \text{1 011011} = \text{complement of } \text{100101} \\
 \text{1 111111 011011} = \text{complement of } \text{000000 100101}
 \end{array}$$

It will be seen that in practice we may increase the number of positions in a negative number merely by adding 1's in as many positions to the left of the original Negative Digit as are required, the new Negative Digit being the left-hand digit.

This, of course, will not affect the value of the number, since:-

$$\text{1 000000000000} + \text{111111000000} = \text{-(000001000000)}$$

Therefore in multiplying 1 011011 by 000101, using the negative number as the multiplicand, we must complete the complement each time a partial product is produced, by adding 1's to convert the partial product into a 13 position number. Thus we have:-

$$\begin{array}{r}
 \text{- 37} \\
 \text{5} \\
 \hline
 \text{-185}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{- +} \\
 \text{1 011011} \\
 \text{+ 000101} \\
 \hline
 \text{1 111111 011011} \\
 \hline
 \text{1 111101 1011} \\
 \text{*1 111101 000111} \\
 \hline
 \text{= -1 + complement of} \\
 \text{000010 111001}
 \end{array}$$

* Note. The 1 produced in the answer to the left of the Negative Digit is ignored and is not treated by the machine as a significant digit.

The partial products are both negative numbers which, when added together, give the correct negative answer.

Contd.....

The final case of multiplication is that of two numbers which are both negative, and is a combination of the previous processes. It might arise in practice when a rebate has to be calculated on goods that have been returned.

Thus $(-37) \times (-5)$ is:-

	+		
		1 011011	
		1 111011	
-	+		
1 111111		011011	
1 111110		11011	
<hr/>			
1 111110		010001	
1 111011		011	
<hr/>			
1 111001		101001	
1 110110		11	
<hr/>			
1 110000		011001	
1 101101		1	
<hr/>			
1 011101		111001	
		100101	Complement of multiplicand
<hr/>			
0 000010		111001	

4.8. Division of numbers in binary form.

Division of one number by another may be carried out by multiplying the dividend by the reciprocal of the divisor. Thus to obtain $x \div y$, first divide 1 by y to obtain the reciprocal and then multiply by x. A programme of orders can be put into the machine so that it can obtain the reciprocal of a number using multiplication and addition only. The machine could, therefore, carry out division without a special Dividing Mechanism in the machine.

But in order to save time and the use of space in the store for holding the programme of orders that would be necessary, a special mechanism is provided to carry out division in a direct fashion.

Division of binary numbers may be carried out just as for long division of decimal numbers, e.g.

5)57(11.4	101)111001(1011.01100	
	<u>101</u>	= $8 + 2 + 1 + \frac{1}{4} + \frac{1}{8}$ etc.
	1000	
	<u>101</u>	= 11.375 etc
	111	
	<u>101</u>	
	1000	
	<u>101</u>	
	110	
	<u>101</u>	
	100	etc.

Contd.....

In order that the machine may carry out this task the procedure has to be carried out rather differently, for the machine when it makes the division in the second step cannot say to itself:-

" 101 into 100 won't go ".

It has to deduct 101 from 100 and establish the fact by obtaining a negative answer. The machine makes use of this negative answer to put a "0" in the appropriate place in the quotient.

Before the machine can proceed to the next step in the division, (i.e. moving the divisor one place to the right and deducting it from the remainder), it must add back the 101 to cancel the previous step.

Thus it has already subtracted 101, and has found that the result is negative:

- 101

It must now

- | | |
|---|--------|
| 1. Add 101 to the remainder to cancel the previous step | + 101 |
| 2. Shift the divisor to give 0101 | |
| 3. Subtract the new divisor from the remainder | - 0101 |

Thus the net result of these three steps is

+ 0101

It is seen therefore that steps 1 and 3 may be combined and the operations are:

- | | |
|-----------------------------------|--------|
| 1. Shift the divisor to give 0101 | |
| 2. Add the new divisor | + 0101 |

The rule is therefore:-

Deduct the divisor, and put "1" in the quotient if the remainder is positive. Otherwise put "0". If the remainder is negative after any step, add the divisor instead of deducting in the next step.

Contd.....

The example worked in this way is:-

101)111001		
<u>-101</u>	1st Remainder 1) Quotient
<u>+100</u>		
<u>-101</u>	2nd Remainder 0	
<u>-10</u>		
<u>+101</u>	3rd Remainder 1	
<u>+111</u>		
<u>-101</u>	4th Remainder 1	
<u>+100</u>		
<u>-101</u>	5th Remainder 0	
<u>-10</u>		
<u>+101</u>	6th Remainder 1	
<u>+110</u>		
<u>-101</u>	7th Remainder 1	
<u>+10</u>		
<u>-101</u>	8th Remainder 0	
<u>-110</u>		
<u>+101</u>	9th Remainder 0	
<u>-1</u>		

etc.

The Dividing mechanism to do this must have circuits that will:-

- (a) move the divisor one place to the right after each step;
- (b) subtract the divisor from the dividend from the remainder if it is positive, but will add the divisor to the remainder if it is negative;
- (c) put 1 in the quotient if the remainder is positive and 0 if the remainder is negative.

.....
Contd.....

5. How the Machine carries out Arithmetic.

Having shown the rules for making certain types of calculation with binary numbers, it is now proposed to show how circuits may be devised to apply these rules within the machine.

5.1. Counting.

Here the problem is to add 1 repeatedly to a previous answer (see para 4.1.). The machine clearly needs two basic storage circuits. The first to store the number 1 (or 00001 if we take five positions), and a second to contain the answer or sub-total after each addition of 1. These circuits are shown as a & t respectively in figure 5.1.1. attached to page 43 .

a is a delay storage circuit with the pulse-train "00001" circulating. This we have termed the "Unity Storage Circuit".

t is a similar delay storage circuit in which the answer has to circulate in step with a. This we have termed the "Sub-total Storage Circuit".

We have to consider how these circuits must be connected in order that the 1 in a may be added into t at each circulation.

At the commencement of the first circulation we have:-

In the Sub-total Circuit <u>t</u>	00000
In the Unity Storage <u>a</u>	<u>00001</u>
The answer required in <u>t</u> is	<u>00001</u>

All that is necessary to obtain this is a "feed line" from circuit a into circuit t so that the pulse in the first position of the former flows into the latter (see figure 5.1.2.). The feed line leads into the sub-total circuit t at the junction j. If now there is a pulse in any binary position of the number in the feed line, or in the sub-total before it reaches j, then the sub-total leaving j will have a pulse in that position.

At the commencement of the second circulation before j is reached, we have:-

In the Sub-total Circuit <u>t</u>	00001
In the Unity Storage <u>a</u>	<u>00001</u>
The answer required in <u>t</u> is	<u>0010</u>

In order to achieve this result the pulse which passes j in the first position of circuit t must be suppressed and a pulse must be supplied to this circuit in the second position.

Contd.....

The machine first needs some device whereby it can detect when there is a pulse in both a and t at the same time. This device takes the form of a "gate", which has two incoming leads; unless there is a pulse on both incoming leads at the same time no pulse will pass along the outgoing lead. Figure 5.1.3. shows leads from circuits a and t into a gate g. The lead from the gate will therefore allow a pulse to pass when there is a pulse in the same position in both a and t (as there is in the circumstance we are at the moment considering). The resulting pulse is first used to suppress the pulse flowing from the junction round circuit t (see figure 5.1.4.) by a connection from g to a "Suppressor", s, which is inserted in circuit t. This Suppressor will not allow a pulse to pass through it from j when there is a pulse in the lead from g. In the first position of the new sub-total, therefore, there will be no pulse.

Next we need to use the pulse from g to carry 1 to the next position. This is done (see figure 5.1.5) by connecting g to a "Delay Mechanism" d, which has the effect of delaying by one position the pulse reaching it.

The delayed pulse now has to be "added" into t in just the same way as the original pulse from a. It is therefore necessary to feed it into the feed line from a through the "Carry Line" (fig 5.1.5).

The pulse in the second position is fed into t and also into g. As no pulse arrives at g from t, g is closed which leaves s open to let through the carry pulse.

The answer circulating in t is therefore now 00010 as required.

At the commencement of the third circulation before j is reached we have:-

In the Sub-total Circuit <u>t</u>	00010
In the Unity Storage <u>a</u>	<u>00001</u>
The answer required in <u>t</u> is	<u>00011</u>

The circuit already provided achieves the required result for, in the first position the pulse from a passes into the circuit t, through j, and in the second position the pulse in t passes round unaltered.

At the commencement of the fourth circulation we have:-

In the Sub-total <u>t</u>	00011
In the Unity Storage <u>a</u>	<u>00001</u>
The answer required in <u>t</u> is	<u>00100</u>

In the first position there is a pulse arriving at g from both the circuit a and the circuit t and so g is open, which means that the suppressor s stops the pulse which has passed through j and the answer is 0 in the first position. The pulse from g also goes to d and produces a carry pulse which is fed back into the feed line from a in the second position.

Contd.....

At g therefore in the second position there is again a pulse both along the feed line from a and from t, and again g is open and s suppresses the pulse passing through j in the second position; the answer produced in t in this position is therefore again 0.

But in the second position there is again a pulse from g which passes to d; so there is a carry pulse to the third position. This time no pulse arrives at g from the circuit t and therefore g is closed and s is open. The carry pulse flows through j into circuit t and produces 1 in the answer.

The answer is therefore 00100 as required.

In the fifth and subsequent circulations the effect is a repetition of previous circulations so that it will be clear that the circuit of figure 5.1.5. does behave in accordance with the rules necessary for counting as set out in 4.1.

These rules are:-

- (a) A pulse in a in any position where there is no pulse in t arriving at j produces a pulse flowing through j into t;
- (b) A pulse in a in a position where there is already a pulse in t arriving at j produces no pulse in t and carries a pulse which is fed back to be added into t through j according to these same rules;
- (c) A pulse in t arriving at j in any position where there is no pulse in a produces a pulse flowing through j into t;
- (d) When no pulse arrives at j from either a or t no pulse flows into t from j.

5.2. Adding

The problem here is to take two numbers and, considering each binary position in turn, to produce the total (see Para. 4.2.). Since each position is dealt with in turn, we need not consider any storage circuit; the two numbers to be added will arrive along two feed lines, and the total will be fed out along another output line to be used or stored elsewhere.

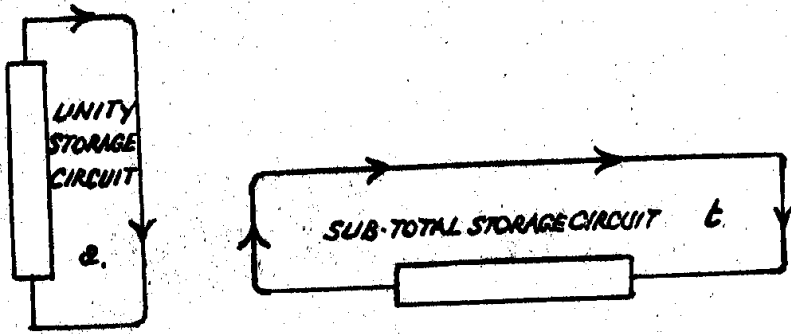
The two numbers used for illustration are those used in Para. 4.2.

	a	01101100
and		
	b	00101010

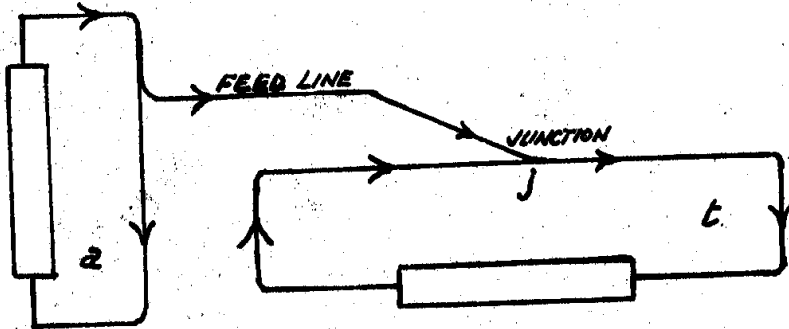
Each binary position is dealt with in turn in the same way as in Section 4.2.

contd....

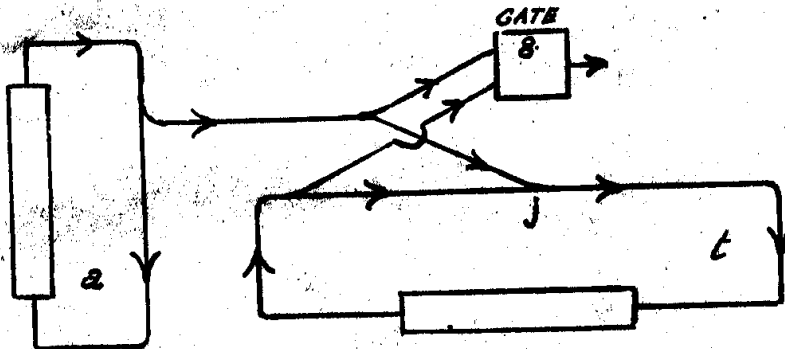
5.1.1.



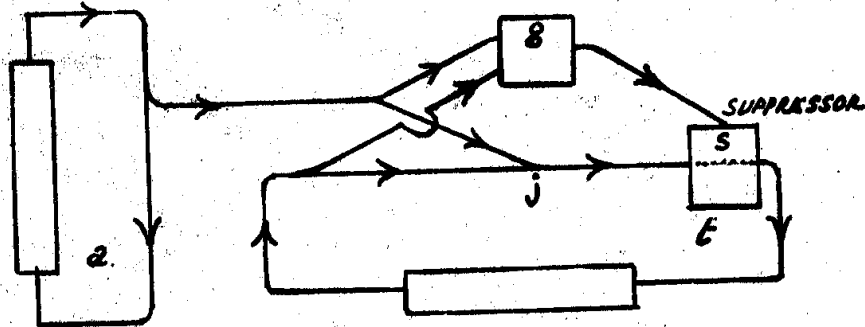
5.1.2.



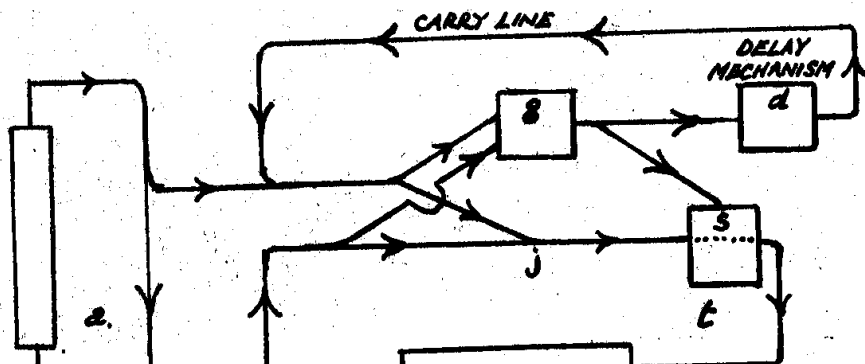
5.1.3.



5.1.4.



5.1.5.



For the first three positions we have-

Arriving through feed line <u>a</u>100
" " " " <u>b</u>010
The answer to be produced in)110
the line <u>t₁</u> must be)	

If we connect the feed lines a and b by means of a junction, j₁, (see figure 5.2.1. attached to page 46) then it will be seen that since in the first position no pulse arrives at j₁, there will be no pulse in the answer, and in the second and third positions the single pulse arriving from a or b will pass through j₁, to give 1 in both positions of the answer.

In the fourth and fifth positions we have:-

Arriving through feed line <u>a</u>01(100)
" " " " <u>b</u>01(010)
The total to be produced in)10(110)
line <u>t₁</u> must be)	

In order to achieve this result the pulse in the fourth position which passes j₁, along t₁, must be suppressed and a pulse created in the fifth position to be added to the 0 in t₁, for this position.

As in the counting circuit (Para 5.1.) we may use a gate, g₁, to detect the presence of a pulse in both a and b simultaneously (see figure 5.2.2). Here when a pulse arrives along neither a nor b, or along only one of them, g₁ produces no pulse; but when a pulse arrives along both a and b a pulse is produced by g₁. A suppresser, s₁, must now be inserted (see figure 5.2.3.) in the line t₁, from j₁, to suppress the pulse which occurs in t₁, from a and b.

To provide the carry pulse one position later, we must also pass the pulse produced by g₁, through a delay mechanism, d₁, (see figure 5.2.4) and along a carry line c₁. We now have the required first total and carry in the lines t₁, and c₁, respectively. These must now be added together to produce the answer.

a	01(100)
b	01(010)
t ₁	00(110)
c ₁	10(000)
t ₂	10(110)

We must, therefore, use a second junction j₂, (see figure 5.2.5), to bring together the numbers in the two lines t₁, and c₁. The total appears in the new output line t₂.

Contd.....

In the sixth, seventh, and eighth positions we have:-

Arriving through feed line <u>a</u>	011(01100)
" " " " <u>b</u>	<u>001(01010)</u>
The first total to be produced) in the line <u>t</u> ₁ must be)	010(00110)
The carry number to be produced) in the line <u>c</u> ₁ must be)	010(10000)
The carry number to be produced) in a second carry line <u>c</u> ₂ must be)	.100(00000)
The final total to be produced in) the output line <u>t</u> ₂ must be)	<u>.100(10110)</u>

The circuit already built up (see figure 5.2.5) will be seen to be sufficient to produce the correct numbers in the lines t₁ and c₁. But in the seventh position there is a pulse in both t₁ and c₁; therefore in the output line t₂, the pulse from j₁ must be suppressed and a new carry pulse must be created in the eighth position. This is done (see figure 5.2.6.) by providing leads from c₁ and t₁ to a gate g₁ which detects the coincident pulses in them. A lead from g₁ to a suppressor s₁ in t₂ suppresses the pulse from j₁; and a lead from g₁ to another delay mechanism d₁ forms the new carry pulse in the eighth position in c₂, which must be added to t₁. This is done by feeding it back into c₁ at the junction j₂ (see figure 5.2.7).

The pulse arriving at j₂ from c₂ passes to j₂ and g₂ where it is added to t₁ by the same process as a carry from c₁, thereby giving the pulse in the output line t₂ which now has the final total 10010110.

NOTE. There can never be a simultaneous pulse arriving along both c₁ and c₂ to be added at the same time to t₁. For there can only be a pulse in c₂ when in the previous position g₁ was open; i.e. when there was then a pulse in t₁. At this time s₁ was open and therefore g₁ was closed so that there could not be a pulse to d₁ to form a carry pulse in c₂.

It can now be seen that the circuit in figure 5.2.7. follows the rules necessary for adding two numbers, as set out in 4.2. These rules are:-

In any position -

- (a) If no pulse arrives along a or b, no pulse appears in t₁.
- (b) If a pulse arrives along either a or b but not both, a pulse appears in t₁.
- (c) If a pulse arrives along both a and b, no pulse appears in t₁ but a pulse is produced in the next position in c₁.

Contd.....

- (d) If there is a pulse in t_1 , but not in c_1 or c_2 , a pulse appears in t_2 .
- (e) If there is no pulse in t_1 , c_1 , or c_2 , no pulse appears in t_2 .
- (f) If there is a pulse in c_1 or c_2 but not in t_1 , a pulse appears in t_2 .
- (g) If there is a pulse in t_1 and in either c_1 or c_2 , no pulse appears in t_2 but a pulse is produced in c_2 to be fed back and dealt with according to the same rules.
- (h) There can never be a pulse in both c_1 and c_2 at the same time.

It will be noticed that the circuit in figure 5.2.7. can be divided into two almost similar halves, and that the second half is identical with the circuit shown for Counting in figure 5.1.5. For this reason the Counting circuit is called a "Half-Adder"; and an Adding Circuit can be made by the appropriate combination of two "Half-Adders".

Contd.....

5.3. Multiplication

As we have seen in 4.3. binary multiplication consists of the production of a series of partial products, which are added as they are produced to give the final product.

If therefore we multiply the two numbers

	a	10110
and	b	1011

we first require to hold the multiplicand a in a form in which it can be used in each stage of the calculation, and also require a circuit to hold the answer. For this purpose we use two Delay Storage circuits, the multiplicand circuit, a, and the total circuit, t. (see 5.3.1. attached to Page 48).

a	10110
b	<u>001</u>
P,	<u>10110</u>
t,	<u>10110</u>

In the first position of the multiplier it is required to use the multiplicand as the partial product if there is a pulse in the multiplier and to feed it into the total circuit, t; or, if there is no pulse in the multiplier, to leave the total circuit unaltered. If we connect the multiplicand circuit a by means of a feed line to an adding unit A.U., which feeds the answer into the total circuit, t, as indicated in 5.2., then the multiplicand will automatically be added into the total circuit (see 5.3.2). We must ensure, however, that this only occurs when there is a pulse in the multiplier. This may be done by inserting what we may call a "Tap" v which is opened by a pulse in the multiplier (see 5.3.3.).

This tap differs from the gate used for counting and adding in that it is opened, not merely for the duration of a single pulse period to allow one pulse to pass when a second pulse arrives simultaneously, but for a number of pulse periods after the operating pulse is received before it is closed again, in order to allow sufficient time for the whole multiplicand to flow through it. The tap is open for the period it takes for the multiplicand and sub-total to flow round their respective storage circuits. The tap is closed at the end of each period of circulation. Each pulse of the multiplier is fed in turn to the tap in successive periods of circulation. If the digit is a 1 the tap opens; otherwise it remains closed.

contd...

a	10110
a ₁	10110.
b	.01.
p ₂	10110.
t ₁	10110
t ₂	<u>1000010</u>

In the second position of the multiplier - it is first necessary to shift the multiplicand one position to the left; in fact it must be done after each circulation of the multiplicand. This process may be carried out by inserting a Delay mechanism, d, in the multiplicand circuit so that after each circulation of the multiplicand every digit occurs one position later (see 5.3.4.).

Since there is a pulse in the multiplier, the tap, v, is open, and the partial product 101100 will flow along the feed line. Since there is already a partial product in the total circuit this second partial product is added to it.

a ₁	10110.
a ₂	10110..
b	.00.
p ₃	00000..
t ₂	1000010
t ₃	<u>1000010</u>

In the third position of the multiplier, the multiplicand has again been delayed one position. Since there is no pulse in the multiplier, v remains closed, and the pulse-train 1011000 will not be allowed to flow to the total circuit.

a ₂	10110..
a ₃	10110...
b	1....
p ₄	10110...
t ₃	1000010
t ₄	<u>11110010</u>

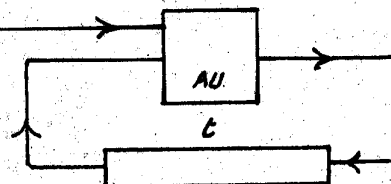
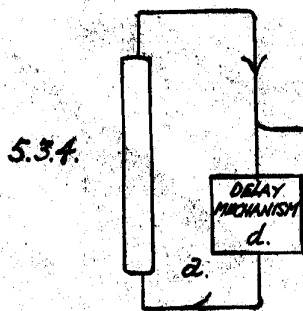
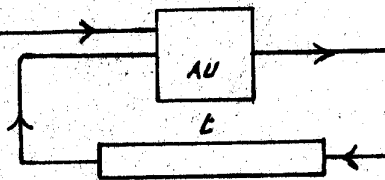
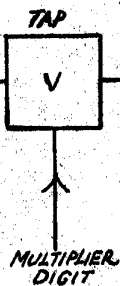
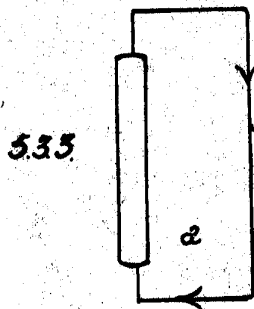
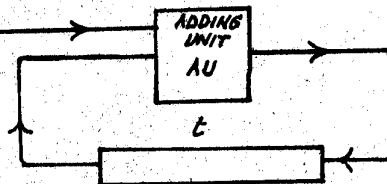
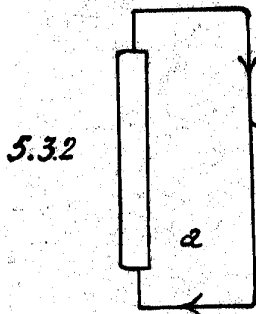
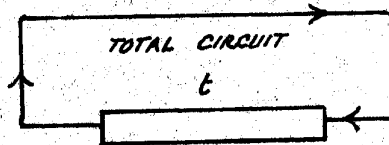
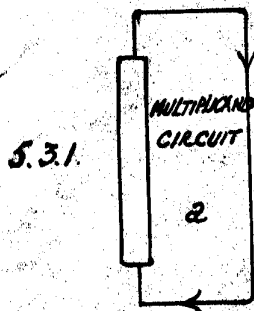
For the fourth position the multiplicand is again delayed one position to give a train of pulses 10110000. v is opened by the pulse in the multiplier to allow this partial product to flow to the Adding Unit where it is added to the previous total to give the final product in the total circuit.

Thus the circuit shown in figure 5.3.4. deals with the multiplier and multiplicand according to the rules of multiplication for binary numbers as set out in 4.3. as follows:-

For each position of the multiplier in turn, starting from the right:-

1. If the multiplier digit is a 1 the multiplicand is added to the answer.
2. If the multiplier digit is a 0 the multiplicand is not added to the answer.
3. It shifts the multiplicand one position to the left.

contd....



5.4. Negative Numbers

It was seen in 4.4. that although in the machine it is not possible to express a negative number as such, there is a single Negative Digit, the 35th, which can be used in conjunction with digits in the 34 positive positions to represent any negative number. Thus -00100101, assuming 8 positive positions only, may be expressed as the Negative Digit -10000 0000 added to the complement of 00100101.

The complement of a number may be defined as the difference between the positive equivalent of the Negative Digit and the number itself. It was shown in 4.4. that if we produce from the original number its Reflected Number:-

Original Number	00100101
Reflected Number	11011010

then the complement can be built up from these two numbers by taking the digits of the original number from the right up to and including the first position that has 1, and for subsequent positions the corresponding digits of the Reflected Number.

We shall now consider a circuit for forming the complement of any number, say 00100101. Referring to figure 5.4.1. attached to page 50, the number is fed in along line a, and we must first form the Reflected Number. We can produce 1's in all positions by feeding a continuous train of pulses, 11111111, along a lead, l. But whenever there is a pulse in the original number we now want 0, and we must suppress the pulses arriving along l in these positions. This is done by inserting a suppressor s in l with a lead from a (see 5.4.1.). The lead from s therefore contains the Reflected Number, r, i.e. 11011010.

But we do not need the digits from this number in the answer until after the first pulse of the original number has passed into the answer; thus we must interpose some device in l which will block the Reflected Number until required. The device needed is a "tap", v, which remains closed until specifically opened, as used in the multiplier circuit (see figure 5.4.2.).

The tap v must be opened immediately after the position of the first pulse in a. If then we provide a lead from a to a delay mechanism, d, connected to v, the first pulse in a will open v at a point in time one pulse interval after the first pulse in a reaches d. (see figure 5.4.3.).

But as soon as the digits of the reflected number, r, are allowed to pass, we no longer need the digits of the original number, and must provide a means of switching off a from the moment that v is open. This can be done by putting a device in a which can be closed by the first pulse from d. Since a must then remain closed till the whole of the number has been dealt with, we will call this device a "Stop Cock", w, (see figure 5.4.4.).

contd....

Finally the output from w and y must be combined by joining the leads from them at the junction j (see 5.4.5.). The resultant pulse train, therefore, corresponds to that in a until the position after the first pulse in a and thereafter to that of the Reflected Number r, i.e. it represents the required Complement.

The circuits in the machine which perform this function are referred to as the Complementer. But the main use of the Complementer is to convert a positive number into a negative number, and for this purpose the Negative Digit must be added to the complement. The original number, and therefore the complement, occupy 34 positions only of the minor cycle; thus in the Complementer at the 35th position of the minor cycle there can be no pulse from a to g in the circuit of figure 5.4.5. This means that g remains open and a pulse passes from l to r. The tap y is also still open, so that a pulse passes along the output lead of the Complementer in the 35th place, following the Complement. Thus the Negative Digit is automatically added to the complement to complete the negative number required.

When feeding negative numbers into the machine it is necessary to include appropriate orders in the programme so that when the minus sign occurs the number is passed through the Complementer before being fed into the store.

5.5. Multiplication of a Negative Number

It was seen in 4.7. that where the multiplicand is negative the normal rules of multiplication are followed, and the multiplying circuit of 5.3.4. can therefore still be used. The circuit, however, requires to be augmented, since each partial product is a negative number. Thus if we multiply -37 by 5, and assume each factor to have six positive binary digits, we get -

```

      1 011011
      000101
  1 111111 011011
  1 111101 1011..
  1 111101 000111

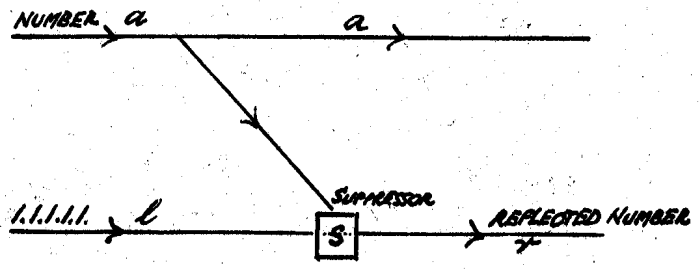
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Since the answer must consist of 12 positive digits, each partial product must also consist of 12 positive digits, although the multiplicand itself has only 6. Therefore in the circuit of 5.3.4. it is necessary at each circulation to examine the negative digit position of the multiplicand and, if it is present, to supply to the partial product the necessary series of pulses for the additional positive positions.

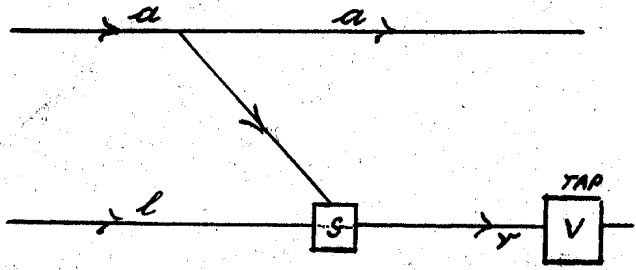
If we examine the multiplication circuit described in 5.3. (repeated as figure 5.5.1. attached to page 51) it will be seen that these additional pulses must be supplied to the feed line which connects the Multiplicand circuit a to the tap y controlled by the multiplier digits.

contd....

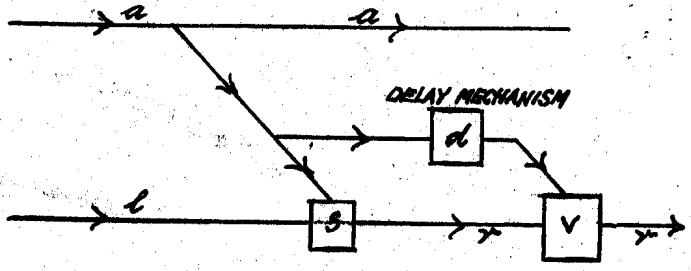
5.4.1



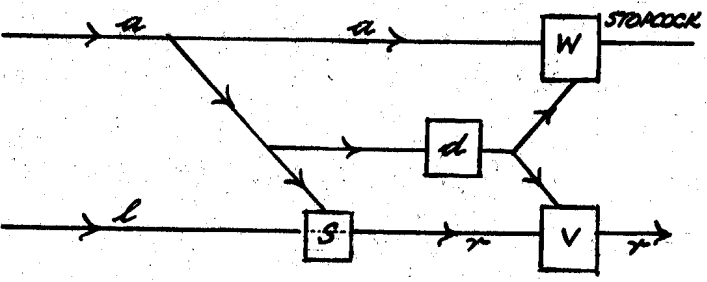
5.4.2



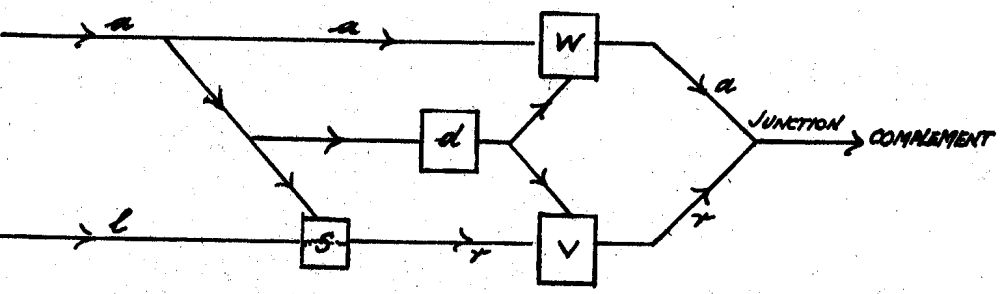
5.4.3



5.4.4



5.4.5



The additional pulses may be made available by connecting to the feed line a lead, $\underline{\ell}$, which carries a continuous pulse-train llllllll (see figure 5.5.2.). But this continuous pulse-train must not be allowed to disturb the partial product flowing from the Multiplicand circuit until the Negative Digit position has arrived; a tap \underline{v} , must therefore be interposed to block them (see figure 5.5.3.).

This tap \underline{v} , is to be opened only when the Negative Digit position of the multiplicand is reached, and we may establish this position by supplying along a lead, \underline{n} , a single pulse corresponding in position to the Negative Digit of the multiplier, which will operate \underline{v} , (see figure 5.5.4.).

But \underline{v} , must be opened only when the multiplicand is negative, and we must therefore ensure that a pulse operates \underline{v} , only when a Negative Digit pulse occurs in the multiplicand. If therefore we insert in \underline{n} a gate, \underline{g} , and feed to it the multiplicand also, then a pulse will arrive at \underline{v} , only when the multiplicand is negative, and only in the negative position (see figure 5.5.5.).

When this happens \underline{v} , is opened and the continuous pulse-train is let through to \underline{v} . The Partial Product with its doubled number of positive positions is thus completed up to and including the new Negative Digit position.

If, and only if, there is a digit in the multiplier for this circulation of the multiplicand, \underline{v} is open, and this partial product is fed to the Adding Unit to be added into the total to date.

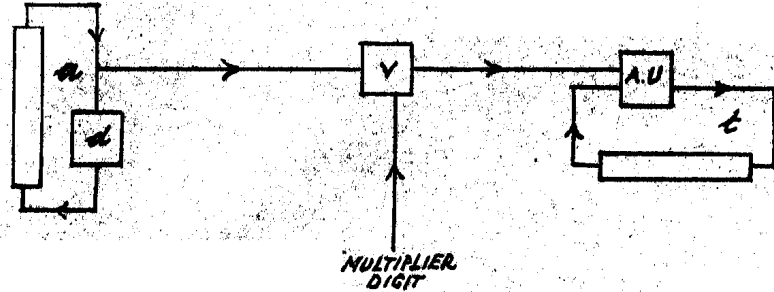
At the end of each complete circulation \underline{v} , is automatically closed again and awaits the Negative Digit before it can be opened again.

This modification to the Multiplication Circuit, therefore, provides for the proper treatment of negative multiplicands.

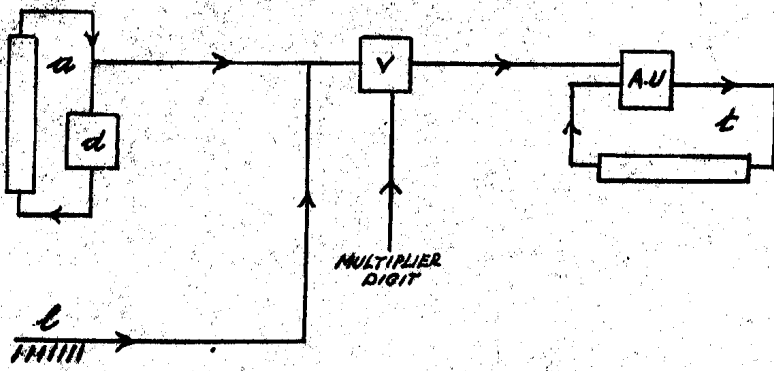
5.6. Other Circuits

Sections 5.1. to 5.5. should serve to illustrate how it is possible to construct circuits which observe arithmetical rules. Fuller particulars of these and other arithmetical circuits used in the machine will be given in Part B, where the parts of the machine will be considered in more detail.

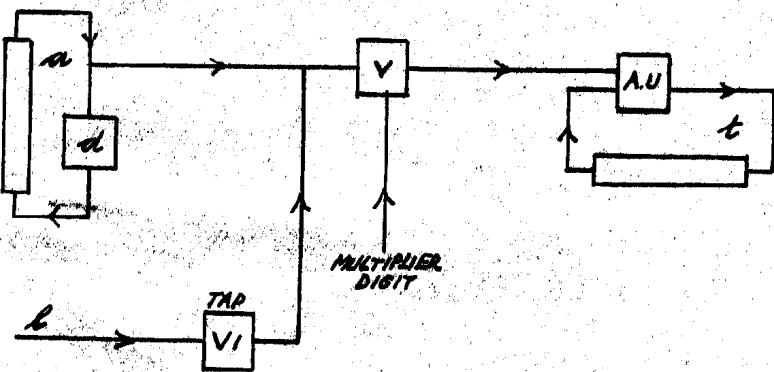
5.5.1



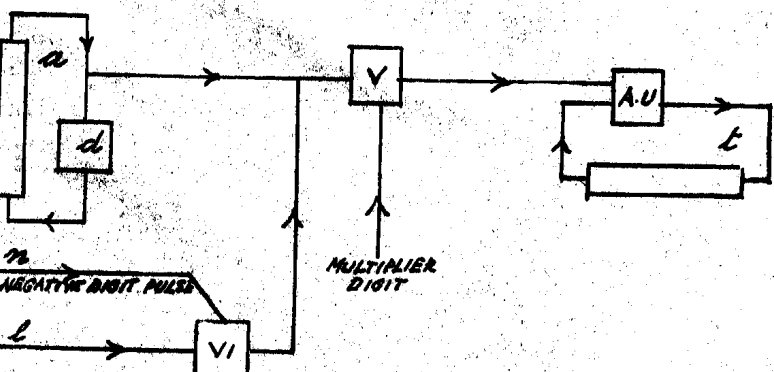
5.5.2



5.5.3



5.5.4



5.5.5

